VIJAYGIRI MATH CLASS

FORMULAE SHEET

ARITHMETIC

NUMBER SYSTEM

1. NATURAL NUMBERS (N) are those numbers which begin from 1 and go up to infinity. [1, 2, 3,…]
2. WHOLE NUMBERS (W) are those numbers which begin from 0 and go up to infinity. [0, 1, 2, 3,…]
3. INTEGERS (Z) are the negative and positive whole numbers and are infinite. [….,-2, -1, 0, 1, 2,…]
4. RATIONAL NUMBERS (Q) are numbers consisting of all integers plus fractions and decimals. These can be represented in the form $\frac{p}{q}$, where q ≠ 0, and can be either terminating or recurring. […,-2, -1.5, -1, - ¼, 0, ½, 1, 2,…]
5. IRRATIONAL NUMBERS (I) are numbers which cannot be expressed in the form $\frac{p}{q}$, where q ≠ 0. These numbers are neither terminating nor recurring. [$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, …]
6. REAL NUMBERS (R) are the union of all Rational and Irrational numbers. They can be represented on a number line.
7. COMPLEX NUMBERS are numbers are of the type a + ib, where a and b are real numbers and i = $\sqrt{-1}$.
8. PRIME NUMBERS are those numbers which are greater than 1 and are divisible only by 1 and the number itself. They have only 2 factors. [2, 3, 5, 7, 11,…]
9. COMPOSITE NUMBERS are numbers which have more than 2 factors. [4, 6, 8, 10,…]
10. PERFECT NUMBERS are those numbers whose sum of all the factors is double the number itself. [6, 28, …]
11. TWIN PRIMES are those prime numbers which differ by 2. [3 and 5, 5 and 7, 11 and 13, …]
12. CO PRIMES are those two numbers which have no common factor except 1. [2 and 3, 3 and 4, 7 and 8, …]
13. ABSOLUTE VALUE of a Real Number is that number which is obtained as a Natural Number. It is denoted by putting the integer between | |. [|-1|= 1, |2| = 2,…]
14. PLACE VALUE of a digit is by virtue of its position in the number.
15. FACE VALUE of a digit is its actual value.
16. STANDARD FORM involves writing large numbers or very small numbers in powers of 10. It is of the form **A x 10^{n}** where ***n*** is a positive or negative and **1 ≤ A < 10.**
17. SIGNIFICANT FIGURES are the number of digits beginning with the digit farthest to the left that is non zero and ending with the digit farthest to the right that is either non zero or zero that is considered to be exact.

RULES OF SIGNIFICANT FIGURES		
1	All non – zero numbers are significant.	**For example 227.61 has 5 significant figures**.
2	All zeroes between two non – zero digits are significant.	**For example 20503 has 5 significant figures**.
3	Leading zeroes are not significant.	**For example 0.53 or 0.0053 has 2 significant figures**.
4	Trailing zeroes in a whole number with a decimal are significant.	**For example 25.00 has 4 significant figures**.
5	Trailing zeroes in a whole number with no decimal are not significant.	**For example 2500 has 2 significant figures**.

18. LOWER BOUND is the smallest value that rounds up to a given number. For example 4.45 is the lower bound of 4.5.
19. UPPER BOUND is the largest value that rounds down to a given number. For example 4.55 is the upper bound of 4.5.
20. ARMSTRONG NUMBERS are those where the sum of the cubes of the digits is equal to the original number. [$153 = 1^3 + 5^3 + 3^3$]
21. FIBONACCI NUMBERS are a series of numbers which follow the rule that from the third number onwards each number is the sum of the two numbers preceding it. [0, 1, 1, 2, 3, 5, …]
22. TRIANGULAR NUMBERS are those numbers which are of the form $\frac{n(n+1)}{2}$, where n is a natural number. [1, 3, 6, …]
23. DIVISION RULE : Dividend = (Divisor x Quotient) + Remainder [27 = 6 x 4 + 3]
24. HCF (Highest Common Factor) of 2 or more numbers is the greatest number that divides each of them exactly.
25. LCM (Lowest Common Multiple) of 2 or more numbers is the smallest number that is divisible by all of them.
26. (HCF of a and b) x (LCM of a and b) = a x b
27. HCF of fractions = $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$
28. LCM of fractions = $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$
29. Sum of first n natural numbers = $\mathbf{\frac{n(n+1)}{2}}$
30. Sum of first n odd numbers = $\mathbf{n^2}$
31. Sum of first n even numbers = n (n + 1)
32. Sum of squares of first n natural numbers = $\mathbf{\frac{n(n+1)(2n+1)}{6}}$
33. Sum of cubes of first n natural numbers = $\left\{\frac{n(n+1)}{2}\right\}^2$
34. BINARY NUMBERS are those numbers which are expressed with a base 2. It has only 2 basic numbers 0 and 1.

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DIVISIBILITY RULES

1. DIVISIBLE BY 2 : If the unit digit of any given number is even than that number is divisible by 2.
2. DIVISIBLE BY 3 : If the sum of all the digits of any given number is a multiple of 3 than that number is divisible by 3.
3. DIVISIBLE BY 4 : If the last 2 digits of any given number is a multiple of 4 than that number is divisible by 4.
4. DIVISIBLE BY 5 : If the unit digit of any given number is 0 or 5 than that number is divisible by 5.
5. DIVISIBLE BY 6 : If a number is divisible by both 2 and 3, than that number is divisible by 6.
6. DIVISIBLE BY 7 : If the unit digit of any given number is doubled and then subtracted from the given number and the answer is 0 or a multiple of 7, than that number is divisible by 7.
7. DIVISIBLE BY 8 : If the last 3 digits of any given number are divisible by 8, than that number is divisible by 8.
8. DIVISIBLE BY 9 : If the sum of all the digits of any given number is a multiple of 9 than that number is divisible by 9.
9. DIVISIBLE BY 10 : If the unit digit of any given number is 0 than that number is divisible by 10.
10. DIVISIBLE BY 11 : If the unit digit of any given number is subtracted from the remaining digits and the answer is 0 or a multiple of 11 than that number is divisible by 11.

SQUARES, CUBES AND SQUARE ROOTS											
n	n^2	n^3	$\sqrt{n}$	**n**	n^2	n^3	$\sqrt{n}$	**n**	n^2	n^3	$\sqrt{n}$
1	1	1	1	**11**	121	1331	3.3166	**21**	441	9261	4.5826
2	4	8	1.4142	**12**	144	1728	3.4641	**22**	484	10648	4.6904
3	9	27	1.7321	**13**	169	2197	3.6056	**23**	529	12167	4.7958
4	16	64	2	**14**	196	2744	3.7417	**24**	576	13824	4.8989
5	25	125	2.2361	**15**	225	3375	3.8729	**25**	625	15625	5
6	36	216	2.4494	**16**	256	4096	4	**26**	676	17576	5.0990
7	49	343	2.6458	**17**	289	4913	4.1231	**27**	729	19683	5.1962
8	64	512	2.8284	**18**	324	5832	4.2426	**28**	784	21952	5.2915
9	81	729	3	**19**	361	6859	4.3589	**29**	841	24389	5.3851
10	100	1000	3.1623	**20**	400	8000	4.4721	**30**	900	27000	5.4772

HCF & LCM

1. FACTOR or factors of a given number are all those numbers that divide the given number completely. Factors of 12 are 1, 2, 3, 6 and 12.

REMEMBER : Factors of any number are always smaller than or equal to the given number.

2. COMMON FACTOR of two or more numbers is that which divides each of them exactly. Example 3 is a common factor of 6, 9, 15
3. HCF (HIGHEST COMMON FACTOR) of two or more numbers is that greatest number which divides all of them exactly.
4. HCF BY PRIME FACTORS : Break up the given numbers into prime factors and then find the product of all the prime factors common to all the numbers. This product is the HCF of the given numbers.

ALGORITHM TO FIND HCF OF TWO OR MORE NUMBERS BY DIVISION	
Step 1	For two given numbers divide the larger number by the smaller
Step 2	Take the remainder.
Step 3	Divide the previous divisor by the remainder.
Step 4	Go on repeating steps 2 and 3 until the remainder is zero
Step 5	The last divisor is the HCF
Step 6	Once the HCF of two numbers is obtained divide the third number by the HCF obtained.
Step 7	Repeat the Steps 2 to 5 till the remainder is zero.
Step 8	The last divisor is the HCF of all the numbers.

5. MULTIPLE of a number is obtained by multiplying the given number by an integer. Multiples of 4 are 4, 8, 12,…, etc.
6. COMMON MULTIPLE of two or more numbers is a number which is completely divisible by all the given numbers. Number 15 is a common multiple of 3 and 5.

REMEMBER : Multiples of any number are always greater than or equal to the given number.

7. LCM (LOWEST COMMON MULTIPLE) of two or more numbers is the smallest number exactly divisible by all the given numbers.
8. LCM BY PRIME FACTORS : Break up the given numbers into prime factors and then find the product of the highest power of all the factors that occur in the given numbers. This product is the LCM of the given numbers.

REMEMBER : Product of any two numbers is equal to the product of their HCF and LCM. **[a x b = HCF x LCM of a and b]**

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ALGORITHM TO FIND LCM OF TWO OR MORE NUMBERS BY DIVISION	
Step 1	Write the given numbers in a row separated by commas.
Step 2	Divide the numbers by the smallest prime number.
Step 3	If any number is not divisible write down the number and continue dividing.
Step 4	Continue dividing by prime numbers until the result in the last row is 1
Step 5	Multiply all the divisors to get the LCM.

FRACTIONS & DECIMALS

1. PROPER FRACTION is that where the numerator is smaller than the denominator.
2. IMPROPER (VULGAR) FRACTION is that where the numerator is greater than the denominator.
3. MIXED FRACTION is that which consists of a whole number and a proper fraction.
4. DECIMAL FRACTIONS are those in which the denominators are in powers of 10.
5. LIKE FRACTIONS are those which have the same denominator.
6. UNLIKE FRACTIONS are those which have different denominators.
7. COMPOUND FRACTION is a fraction of a fraction.
8. COMPLEX FRACTION is that in which either the numerator or the denominator or both are fractions.

NOTE

1. A fraction can be terminating, recurring or non – terminating, non – recurring decimal.
2. A fraction whose denominator consists of only 2^n or 5^n or ($2^n \times 5^n$) will be a terminating decimal where $n \in N$.
3. A decimal number can be converted to a fraction by removing the decimal and putting the denominator as 10^n where n will depend on the number of digits there are after the decimal.

STEPS TO CONVERT A RECURRING DECIMAL TO A FRACTION			
If the number is $0.\bar{a}$		**If the number is $0.\overline{ab}$**	
Step 1	Write $x = 0.\bar{a}$	Step 1	Write $x = 0.\overline{ab}$
Step 2	Multiply both sides by 10 to get $10x = a.\bar{a}$	Step 2	Multiply both sides by 100 to get $100x = ab.\overline{ab}$
Step 3	Subtract first from second to get $9x = a$	Step 3	Subtract first from second to get $99x = ab$
Step 4	Divide both sides by 9 to get $x = \frac{a}{9}$	Step 4	Divide both sides by 99 to get $x = \frac{ab}{99}$
Step 5	Thus $0.\bar{a} = \frac{a}{9}$	Step 5	Thus $0.\overline{ab} = \frac{ab}{99}$
If the number is $0.a\bar{b}$		**If the number is $a.\overline{bc}$**	
Step 1	Write $x = 0.a\bar{b}$	Step 1	Write $x = a.\overline{bc}$
Step 2	Multiply both sides by 10 to get $10x = a.\bar{b}$	Step 2	Multiply both sides by 100 to get $100x = abc.\overline{bc}$
Step 3	Again multiply both sides by 10 to get $100x = ab.\bar{b}$	Step 3	Subtract first from second to get $99x = abc$
Step 4	Subtract second from third to get $90x = ab$	Step 4	Divide both sides by 99 to get $x = \frac{abc}{99}$
Step 5	Divide both sides by 90 to get $x = \frac{ab}{90}$	Step 5	Thus $a.\overline{bc} = \frac{abc}{99}$
Step 6	Thus $0.a\bar{b} = \frac{ab}{90}$		

RATIO & PROPORTION

1. RATIO is the number of times one quantity contains another quantity of the same kind and is written as $a : b$.
2. ANTECEDENT is the first term of the ratio and CONSEQUENT is the second term of the ratio.

1	Duplicate Ratio of $a : b$ is $a^2 : b^2$	3	Sub Duplicate Ratio of $a : b$ is $\sqrt{a} : \sqrt{b}$	5	Reciprocal Ratio of $a : b$ is $\frac{1}{a} : \frac{1}{b}$
2	Triplicate Ratio of $a : b$ is $a^3 : b^3$	4	Sub Triplicate ratio of $a : b$ is $\sqrt[3]{a} : \sqrt[3]{b}$	6	Compound Ratio of a : b & c : d is ac : bd
Theorem	If the ratio between the first and second quantities is $a : b$ and the ratio between the second and third quantities is $c : d$ then the ratio between the first, second and third quantities is $ac : bc : bd$.				

3. PROPORTION is there when four quantities are such that the product of the extremes is equal to the product of the means. [a, b, c, d are said to be in proportion if $ad = bc$]

4. CONTINUED PROPORTION is there when the ratio of the first to the second is equal to the ratio of the second to the third. [a, b, c are said to be in continued proportion if $a : b = b : c$]

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TYPES OF PROPORTION	
1	**Invertendo Rule** states that if four quantities *a, b, c, d* are in proportion such that $a : b = c : d$, then $b : a = d : c$.
2	**Altertendo Rule** states that if four quantities *a, b, c, d* are in proportion such that $a : b = c : d$, then $a : c = b : d$.
3	**Componendo Rule** states that if four quantities *a, b, c, d* are in proportion such that $a : b = c : d$, then $\frac{a+b}{b} = \frac{c+d}{d}$
4	**Dividendo Rule** states that if four quantities *a, b, c, d* are in proportion such that $a : b = c : d$, then $\frac{a-b}{b} = \frac{c-d}{d}$
5	**Componendo and Dividendo Rule** states that if *a, b, c, d* are in proportion such that $a : b = c : d$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

NOTE

1. If three quantities *a, b, c* are given then the fourth proportional is $x = \frac{bc}{a}$
2. If two quantities *a, b* are given then the third proportional is $x = \frac{b^2}{a}$
3. If two quantities *a, b* are given then the mean proportional is $x = \sqrt{ab}$

Theorem 1	If in a mixture of *x* litres of milk and water the ratio of milk to water is *a : b*, then the quantity of water to be added to the mixture to make the ratio *c : d* is $\frac{x(ad - bc)}{c(a+b)}$
Theorem 2	In a mixture containing milk and water in the ratio a : b, if x litres of water is added to make the ratio of milk to water as a : c, then the quantity of milk is $\frac{ax}{c-b}$ and of water is $\frac{bx}{c-b}$
Theorem 3	If two quantities A and B are in the ratio x : y, then $\frac{A+B}{A-B} = \frac{x+y}{x-y}$
Theorem 4	If two quantities are in the ratio *a : b* a number x is added so that the ratio becomes *c : d* then $x = \frac{ad - bc}{c - d}$
Theorem 5	If two quantities are in the ratio *a : b* a number x is subtracted so that the ratio becomes *c : d* then $x = \frac{bc - ad}{c - d}$

VARIATION

1. VARIATION is a relationship between two variables such that a change in one brings about a change in the other.
2. DIRECT VARIATION is that in which an increase or decrease in one variable brings about an increase or decrease in the other variable. [$a \propto b \Rightarrow a = kb$, where k is a constant]
3. INDIRECT VARIATION is that in which an increase or decrease in one variable brings about a decrease or increase in the other variable. [$a \propto \frac{1}{b} \Rightarrow a = \frac{k}{b}$, where k is a constant]

ALGORITHM FOR VARIATION PROBLEMS	
Step 1	Write the equation of the variation. For direct variation write y = kx and for indirect variation write $y = \frac{k}{x}$
Step 2	Substituting the values of x and y from one statement, find the value of k
Step 3	Substituting this value of k and the value of the given variable, find the value of the other variable.

NOTE : 1. Cost always varies directly to quantity 2. Work always varies indirectly to number of men or days.

PARTNERSHIP

1. PARTNERSHIP is an association of two or more persons who invest their money together in a business.
2. SIMPLE PARTNERSHIP is one in which the capital of the partners is invested for the same time period.
3. COMPOUND PARTNERSHIP is one in which the capital of the partners is invested for the different time periods.
4. DISTRIBUTION OF PROFIT = $\frac{\text{Amount invested by one partner}}{\text{Total Amount invested by all}} \times 100$

NOTE

1. In compound partnership, the ratio of profits is directly proportional to both money and time so they are multiplied together to get the corresponding shares. If investments are in the ratio a : b : c and time for which the investments are made is in the ratio x : y : z then the ratio of their shares will be ax : by : cz
2. If investments are in the ratio a : b : c and profits are in the ratio x : y : z, then the ratio of the time is $\frac{x}{a} : \frac{y}{b} : \frac{z}{c}$

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SPEED, TIME & DISTANCE

1	**Distance = Speed x Time**	2	**Speed =** $\frac{\text{Distance}}{\text{Time}}$	3	**Time =** $\frac{\text{Distance}}{\text{Speed}}$
4	If the speed is changed in the ratio $a : b$ then the time taken changes in the ratio $b : a$.				
5	To convert the speed from km per hour to metres per second multiply the given speed by $\frac{5}{18}$.				
6	To convert the speed from metres per second to km per hour multiply the given speed by $\frac{18}{5}$.				
7	If a distance is covered at a speed of x km/hr and the same is covered at a speed of y km/hr, then **average speed =** $\frac{2xy}{x+y}$ **km/hr**.				
8	Relative Speed of 2 vehicles with speeds S_1 and S_2 moving in opposite directions is $S_1 + S_2$.				
9	Relative Speed of 2 vehicles with speeds S_1 and S_2 moving in same direction is $S_1 - S_2$.				
10	Relative Speed of a boat moving downstream = Speed of boat + Speed of river.				
11	Relative Speed of a boat moving upstream = Speed of boat – Speed of river.				
12	Time taken by 2 trains of different lengths moving in opposite directions with speeds S_1 and S_2 is $\frac{\text{Sum of lengths of trains}}{S_1 + S_2}$				
13	Time taken by 2 trains of different lengths moving in same direction with speeds S_1 and S_2 is $\frac{\text{Sum of lengths of trains}}{S_1 - S_2}$				
14	Distance between two cities A and B when trains starting from each city and moving with speeds S_1 and S_2 meet and the difference in distance covered is given then **Distance = Difference in distance covered x** $\frac{S_1 + S_2}{S_1 - S_2}$				
15	Distance when 2 different speeds and arrival times are given $= \frac{\text{Product of 2 speeds}}{\text{Difference of 2 speeds}}$ **x Difference between arrival time**.				
16	Distance when 2 different speeds and total time is given $= \frac{\text{Product of 2 speeds}}{\text{Addition of 2 speeds}}$ **x Total time**.				
17	If 2 persons start at the same time from 2 points in opposite directions and take a and b hours to complete the journey then their speeds are in the ratio $\sqrt{b} : \sqrt{a}$.				
18	If a certain distance is covered at x km per hour and the same distance is covered at y km per hour then the average speed during the whole journey = $\frac{2xy}{x+y}$ km per hour.				
19	When two different speeds are given for the same distance and the difference in arrival time is known then the **Required Distance =** $\frac{\text{Product of 2 speeds}}{\text{Difference of 2 speeds}}$ **x Difference in Arrival Time.**				
20	When half of the journey is covered at a km per hour and the remaining half at b km per hour in T hours then the **Distance =** $\frac{2 \times T \times a \times b}{a+b}$ km.				
21	When on increasing the speed time taken to cover a distance decreases and on decreasing the speed time taken to cover the same distance increases then **Speed =** $\frac{2 \times \text{Increase in speed} \times \text{Decrease in speed}}{\text{Increase in speed} - \text{Decrease in speed}}$ **km per hour**.				

TIME & WORK

1	**Work done = Time taken x Rate of work**	2	**Time taken =** $\frac{1}{\text{Rate of work}}$	3	**Rate of work =** $\frac{1}{\text{Time taken}}$
Theorem 1	If A can do a piece of work in x days and B can do it in y days, then both of them together will do it in $\frac{xy}{x+y}$ days.				
Theorem 2	If A and B together can do a piece of work in x days and A can do it in y days, then B will do it in $\frac{xy}{y-x}$ days.				
Theorem 3	If A, B and C can do a piece of work in x, y and z days respectively together they will do it in $\frac{xyz}{xy+yz+xz}$ days.				
Theorem 4	If A and B together take x days finish a work and A alone takes y days then B alone will take $\frac{xy}{y-x}$ days.				
Theorem 5	If tap A fills a tank in x hours and tap B empties it in y hours then time taken to fill the tank is $\frac{xy}{y-x}$ hours.				
Theorem 6	If taps A and B take x and y hours to fill the tank and C takes z hours to empty it then time taken to fill the tank is $\frac{xyz}{yz+xz-xy}$ hours.				

ALGORITHM FOR TIME & WORK PROBLEMS	
Step 1	Identify the total work that needs to be completed.
Step 2	For each group or individual determine the time taken to complete the work
Step 3	Calculate their rate of work by taking the reciprocal of their time.
Step 4	If multiple individuals are working add their individual rates to get the combined rate of the group.
Step 5	To find the time needed to complete a specific task with a given group use the formula Time = $\frac{\text{Work}}{\text{Combined Rate}}$

REMEMBER	1. Sometimes people with different efficiencies will come together to finish a task, At this time adjust their rate of work based on their relative efficiency. 2. Set up ratios to compare the work done by different individuals or groups in a given time frame.

FORMULAE SHEET

PROFIT & LOSS

1. COST PRICE (C. P) is the amount of money paid to buy a product or service.
2. FIXED COST is a constant cost that does not change. It is mostly applicable to service bought
3. VARIABLE COST is that which changes with the number of units bought.
4. SELLING PRICE (S. P) is the price at which the product or service is offered to the buyer or customer.
5. PROFIT is made the selling price is greater than the cost price. **S. P > C. P**
6. LOSS is made when the selling price is less than the cost price. **S. P < C. P**
7. MARKED PRICE is the price at which a seller intends to sell before any discounts and is normally higher than the cost price.
8. DISCOUNT is the amount subtracted from the marked price before selling it.

IMPORTANT RESULTS					
1	Profit = S. P – C. P	2	Loss = C. P – S. P	3	Discount = M. P – S. P
4	Profit % = $\frac{\text{Profit x 100}}{\text{C.P}}$	5	Loss % = $\frac{\text{Loss x 100}}{\text{C.P}}$	6	Discount % = $\frac{\text{Discount x 100}}{\text{Marked Price}}$
7	C. P = S. P x $\frac{100}{100 + P\,\%}$ or $\frac{100}{100 - L\,\%}$	8	S. P = C. P x $\frac{100 + P\,\%}{100}$ or $\frac{100 - L\,\%}{100}$	9	S. P = M. P x $\frac{100 - \text{Discount}\,\%}{100}$
10	Error in weight = True Weight – False Weight	11	Gain % in Error = $\frac{\text{Error}}{\text{False Weight}}$ x 100	12	M. P = $\frac{100 \text{ x S.P}}{100 - \text{Discount}\,\%}$
13	Gain % or Loss % when y gram is used for 1 kg and sold at x % gain or loss on C. P				$(100 \pm x)\left(\frac{1000}{y}\right) - 100$
14	Resultant Profit % when there are two successive profits of x% and y%				$\left(x + y + \frac{xy}{100}\right)$
15	If **x** part of a commodity is sold at *a*% profit, **y** part is sold at *b*% profit, **z** part is sold at *c*% profit and the total profit is **P**, then the value of the total consignment is				$\frac{100\,P}{ax + by + cz}$
16	When each of the two commodities is sold at the same price, making a profit of x% on one and a loss of y% on the other then the total gain or loss percentage is				$\frac{100\,(x - y) - 2xy}{(100 + x) + (100 - y)}$

PERCENTAGE

1. PERCENTAGE is a fraction with denominator 100.
2. TO CONVERT A FRACTION INTO A PERCENTAGE multiply by 100.
3. TO CONVERT A PERCENTAGE INTO A FRACTION divide by 100 and reduce it
4. TO CONVERT A DECIMAL INTO A PERCENTAGE move the decimal 2 places to the right.
5. TO CONVERT A PERCENTAGE INTO A DECIMAL move the decimal 2 places to the left.
6. To find x % of a certain number multiply x with the number and divide the result by 100.
7. PERCENTAGE DIFFERENCE $= \frac{\textbf{Absolute Difference}}{\textbf{Average}}$ **x 100**
8. PERCENTAGE INCREASE or DECREASE $= \frac{\textbf{Final Value} - \textbf{Initial Value}}{\textbf{Initial Value}}$ **x 100** OR $\frac{\textbf{Initial Value} - \textbf{Final Value}}{\textbf{Initial Value}}$ **x 100**

Theorem 1	If two values are respectively x% and y% more than a third value, then the first is $\frac{100 + x}{100 + y}$ **x 100 % of the second.**
Theorem 2	If A is x% of C and B is y% of C, then A is $\frac{x}{y}$ **x 100 % of B**
Theorem 3	If after spending x% of one's money as one expense and y% of the same money as second expense if money is left then **Initial Money** = $\frac{\text{Money left x 100 x 100}}{(100 - x)\,(100 - y)}$
Theorem 4	If x% of a quantity is taken by the first, y% of the remaining is taken by the second, z% of the remaining is taken by the third and if A is left then **Initial Money** = $\frac{A \text{ x } 100 \text{ x } 100 \text{ x } 100}{(100 - x)\,(100 - y)\,(100 - z)}$
Theorem 5	If x% of a quantity is added, y% of the increased quantity is added, z% of the increased quantity is added and if A is the final amount then **Initial Quantity** = $\frac{A \text{ x } 100 \text{ x } 100 \text{ x } 100}{(100 + x)\,(100 + y)\,(100 + z)}$
Theorem 6	If the population of a place increases by x% in one year, y% in the second year and z% in the third year then the Present Population of the place = $\frac{\text{Present Population x }(100 + x)\,(100 + y)\,(100 + z)}{100 \text{ x } 100 \text{ x } 100}$
Theorem 7	If the price of a commodity increases by r%, then the reduction in consumption needed so as not to increase the expenditure is $\frac{r}{100 + r}$ **x 100 %**
Theorem 8	If the price of a commodity decreases by r%, then the increase in consumption needed so as not to decrease the expenditure is $\frac{r}{100 - r}$ **x 100 %**
Theorem 9	If the value of A is r% more than B then B is $\frac{r}{100 + r}$ **x 100 %** less than A.
Theorem 10	If the value of A is r% less than B then B is $\frac{r}{100 - r}$ **x 100 %** more than A.
Theorem 11	If the value is first increased by x% and then decreased by x%, then the net change is always a decrease which is $\frac{x^2}{100}$
Theorem 12	If the value is first increased by x% and then decreased by y% then there is $\left(x - y - \frac{xy}{100}\right)$ % increase or decrease depending if the sign is positive or negative.

CALENDAR & CLOCK

1. LEAP YEAR is that which is divisible either by 4 or by 400.
2. CENTURY LEAP YEAR is that which is divisible only by 400.
3. ODD DAY OR DAYS is that additional day or days left in a year after 52 weeks.

DETERMINING A LEAP YEAR		
1	Any year that is not divisible by 4 is not a leap year.	[1962, 1982, 2007, etc.]
2	Any year that is divisible by 4 but not by 100 is a leap year.	[1964, 1988, 2004, etc.]
3	Any year that is divisible by 400 is a leap year.	[1600, 2000, 2400, etc.]
4	Any year that is divisible by 100 but not by 400 is not a leap year.	[1700, 1900, 2200, etc.]

NUMBER OF ODD DAYS		
1	In an ordinary year	**1**
2	In a leap year	**2**
3	In 100 years	**5**
4	In 200 years	**3**
5	In 300 years	**1**
6	In 400 years	**0**

ALGORITHM FOR A CALENDAR PROBLEM	
Step 1	Determine the nearest century year before the given year.
Step 2	Count the number of years after the century year obtained in Step 1.
Step 3	For every 100 years determine the number of odd days as in the table above.
Step 4	Next calculate the number of leap years and the ordinary years.
Step 5	Multiply the number of leap years with 2 and add the number of ordinary years.
Step 6	Divide the answer obtained in Step 5 by 7 and note the remainder. This again is the number of odd days.
Step 7	For the current year add up all the days till the given date and divide be 7. Remainder is number of odd days.
Step 8	Add up all the odd days. If the number is greater than 7 divide by 7 and note the remainder.
Step 9	Taking Monday as the first day identify the day for the remainder obtained in step 8.

1. Angle tuned by an hour hand in 1 minute = $\frac{1}{2}^{\circ}$
2. Angle turned by a minute hand in 1 minute = 6°
3. Relative angular speed of a minute hand with respect to the hour hand = 5.5° per minute.
4. Angle between the hands of the clock at H hours and M minutes = $\left|30\,\mathrm{H} - \frac{11}{2}\,\mathrm{M}\right|$

SOME FACTS ABOUT CLOCK	
1	Both the hands of the clock coincide 11 times in 12 hours.
2	Both the hands meet after every $65\frac{5}{11}$ minutes.
3	Both the hands are opposite to each other (at 180°) 11 times in 12 hours.
4	Both the hands are at right angles to each other (90°) 22 times in 12 hours.
5	Any other angle between 0° and 180° between the two hands is made 22 times in 12 hours.

ALGORITHM FOR CLOCK PROBLEMS		
1	To find when the two hands meet.	1. Determine the angle between the two hands at the given time. 2. Divide the angle by 5.5 to get the minutes
2	To find when the hands are at right angles.	1. In any given 1 hour the two hands will be at 90° twice. 2. Determine the angle between the two hands at the given time. 3. Subtract and add θ to the angle in step 2 to make it 90° 4. Divide both values of Step 3 by 5.5 to get the time.
3	To find the time at A hours when a clock gains x second in y minutes and was set right at B hours.	1. Find the seconds gained in 1 hour. 2. Multiply the seconds gained in 1 hour by (B – A). 3. Divide the answer in Step 2 by 60 to get the minutes. 4. Add the answer of Step 3 to B
4	To find when a clock which loses x minutes in *1* hour will show the correct time again if it was set right at A hours.	1. For the clock to show the correct time again it should lose 12 hours. 2. Find the minutes lost in 12 hours. 3. Multiply answer of Step 3 by 12. 4. Divide the answer by 24 to know when the day and time when the correct time will be shown again.
5	To know the correct time at B hours when a clock which gains x second in y minutes was set right at A hours.	1.

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FORMULAE SHEET

SIMPLE INTEREST

P is the Principal sum lent, I is the Interest money paid by the borrower, R is the Rate of interest charged for 1 year
T is the time in years for which the money is borrowed or lent, A is the total sum of money paid back by the borrower.

$I = \frac{PRT}{100}$	$R = \frac{I \times 100}{PT}$	$T = \frac{I \times 100}{PR}$	$P = \frac{I \times 100}{RT}$	$P = \frac{A \times 100}{100 + RT}$	$A = P + I$	$A = P\left(\frac{100 + RT}{100}\right)$

1.	If time is given as months then divide the time given by 12.
2.	If time is given as days then divide the time given by 365.
3.	The annual payment that will discharge a debt of Rs.A due in t years at r % per annum is = $\frac{100A}{100t + \frac{rt(t-1)}{2}}$
4.	If a sum of money was lent out in n parts in such a way that the interests on different parts at different rates and for different time periods are all equal then the ratio in which the sum was divided is $\frac{1}{r_1t_1} : \frac{1}{r_2t_2} : \frac{1}{r_3t_3} : \ldots \frac{1}{r_nt_n}$
5.	If two equal amounts are deposited at different rates for different time periods and the difference between their interests is given then **Sum** = $\frac{(\text{Difference in Interest}) \times 100}{\lvert r_1t_1 - r_2t_2 \rvert}$

COMPOUND INTEREST

1	$C.I = P\left\{\left(1 + \frac{R}{100}\right)^N - 1\right\}$ for interest calculated annually.	5	$A = P\left(1 + \frac{R}{100}\right)^N$ for interest calculated annually
2	$C.I = P\left\{\left(1 + \frac{R}{200}\right)^{2N} - 1\right\}$ for interest calculated half yearly.	6	$A = P\left(1 + \frac{R}{200}\right)^{2N}$ for interest calculated half yearly.
3	$C.I = P\left\{\left(1 + \frac{R}{400}\right)^{4N} - 1\right\}$ for interest calculated quarterly.	7	$A = P\left(1 + \frac{R}{400}\right)^{4N}$ for interest calculated quarterly
4	$C.I = P\left\{\left(1 + \frac{R}{1200}\right)^{12N} - 1\right\}$ for interest calculated monthly.	8	$A = P\left(1 + \frac{R}{1200}\right)^{12N}$ for interest calculated monthly.

9	$A = P\left(1 + \frac{R_1}{100}\right)^N \left(1 + \frac{R_2}{100}\right)^N$ …. for changing rates of interest calculated annually
10	$P = A\left(\frac{100}{100 + r_1}\right)\left(\frac{100}{100 + r_2}\right)\left(\frac{100}{100 + r_3}\right)$… when amount with changing rates of interest is given.
11	**Rate of Interest** = $\frac{I_2 - I_1}{I_1}$ **or** $\frac{A_2 - A_1}{A_1}$ when compound interest or amounts of 2 successive years is given.
12	**Rate of interest** = $\frac{2\,(C.I - S.I)}{S.I} \times 100$, when simple interest and compound interest on a sum of money is given
13	**Principal = (C.I – S.I) x** $\left(\frac{100}{r}\right)^2$, when the difference between C.I and S.I on a certain sum for 2 years at r% is given.
14	**Difference = P x** $\left(\frac{r}{100}\right)^2$, when the difference between C.I and S.I on a certain sum for 2 years at r% is to be found.
15	**Difference** = $\frac{P \times r^2 (300 + r)}{100^3}$, when the difference between C.I and S.I on a certain sum for 3 years at r% is to be found.
16	$C.I = S.I + \frac{r \times S.I}{2 \times 100}$ when S. I at r% for 2 years is given and C. I on the same sum at the same rate and for the same time is to be found.
17	Depreciated Value : $V_n = V_o\left(1 - \frac{R}{100}\right)^n$, where V_n = Present value, V_o = Original value, r = rate of depreciation.
18	Present value of a sum due in n years at r% compound interest is **P.V.** = $P\left(\frac{100}{100 + r}\right)$
19	EQUAL INSTALMENTS WITH C. I : **Loan Amount** = $P\left[\left(\frac{100}{100 + r}\right) + \left(\frac{100}{100 + r}\right)^2 + \left(\frac{100}{100 + r}\right)^3 + \cdots + \left(\frac{100}{100 + r}\right)^n\right]$ where P is each equal instalment, r is the rate of interest per annum and n is the number of years.
20	AMOUNT OF REGULAR ANNUITY is $A = R\left[\frac{(1 + i)^n - 1}{i}\right]$ where R is the periodic payment payable, n is number of periods and $i = \frac{r}{100}$
21	PRESENT VALUE OF REGULAR ANNUITY is $P = R\left[\frac{1 - (1 + i)^{-n}}{i}\right]$ where R is the periodic payment payable, n is number of periods and $i = \frac{r}{100}$
22	FUTURE VALUE OF ANNUITY DUE is $A = R(1 + i)\left[\frac{(1 + i)^n - 1}{i}\right]$ where R is the periodic payment payable, n is number of periods and $i = \frac{r}{100}$
23	PRESENT VALUE OF ANNUITY DUE is $P = R(1 + i)\left[\frac{1 - (1 + i)^{-n}}{i}\right]$ where R is the periodic payment payable, n is number of periods and $i = \frac{r}{100}$
24	PRESENT VALUE OF DEFERRED ANNUITY is $P = R(1 + i)^{-m}\left[\frac{1 - (1 + i)^{-n}}{i}\right]$ or $P = \frac{R}{i}\left[\frac{1}{(1 + i)^m} - \frac{1}{(1 + i)^{m+n}}\right]$ where m is the intervals for which the annuity is deferred, n is the number of periods R is periodic payment payable, $i = \frac{r}{100}$

FORMULAE SHEET

GOODS & SERVICE TAX (GST)

1. DEALER is any person or company who buys goods or services for resale.
2. INTRA STATE SALES is the sale of goods and services within the same state or union territory.
3. INTER STATE SALES is the sale of goods and services outside the state or union territory.
4. INPUT TAX is the GST paid by the dealer.
5. OUTPUT TAX is the GST collected by the dealer.

TYPES OF TAXES UNDER GST :

(i) CGST is Central Goods and Services Tax levied on all intra state sales and is 50% of the GST collected and it goes to the Central Government.

(ii) SGST is State Goods and Services Tax levied on all intra state sales and is 50% of the GST collected and it goes to the State Government.

(iii) IGST is Integrated Goods and Services Tax levied on all inter – state sales, on all import and export of goods and services into India and outside India and it goes fully to the Central Government.

ALGORITHM FOR COMPUTING GST	
Step 1	Calculate the GST paid by the dealer on his purchase (Input GST)
Step 2	Compute the selling price of the dealer (purchase price + profit)
Step 3	Calculate the GST collected by the dealer on his selling price (Output GST)
Step 4	GST paid by the dealer is Output GST – Input GST.
Step 5	For all intra state sales CGST = SGST = 50 % of GST

INCOME TAX

1. FINANCIAL YEAR is a period of twelve months used by governments, business and other organizations to calculate their budgets, profits and losses.
2. ASSESSMENT YEAR is the year immediately after the Financial Year.
3. GROSS INCOME is the total income of an individual earned from different sources in a Financial Year.
4. EXEMPTIONS are certain types of income that are not taxable. In India tax on agricultural income, gifts, inheritances, awards, interest on PPF, etc. are exempted.
5. DEDUCTION is a provision in income tax that allows an individual to reduce the tax liability on the total income. The various types of deductions are House Rent Allowance (HRA), Standard Deduction of Rs.50 000.00 allowed for salaried or individual person and Deductions under 80C, 80D, 80E, 80G and under 80TTA.
6. TAXABLE INCOME is the income used to calculate the income tax in a given financial year after subtracting all exemptions and deductions allowed from the gross income.
7. INCOME TAX SLAB is used to calculate the income tax where different rates of interest are prescribed for different slabs.
8. CESS is a form of tax collected by the government for the development of certain sectors like Health, Education.

ALGORITHM TO CALCULATE INCOME TAX	
Step 1	Find the Gross Income.
Step 2	Subtract HRA if eligible and Standard Deduction from the Gross Income.
Step 3	Subtract all deductions under Section 80 C subject to its maximum limit.
Step 4	Subtract all deductions under Section 80 D subject to its maximum limit.
Step 5	Subtract all deductions under Section 80 E subject to its maximum limit.
Step 6	Calculate the income tax based on the balance in Step 3 as per the income tax slab.
Step 7	Subtract tax relief if eligible under section 87A from the income tax obtained in Step 6.
Step 8	Add the required cess to the amount obtained in Step 7.

UTILITY BILLS

1. UTILITY BILL is a detailed invoice issued and paid once a month for utilities such as electricity, water, gas, etc.
2. TARIFF RATE is a price assigned to different utilities by the government. It consist of a Fixed Charge irrespective of the consumption and a Variable Charge which depends on the consumption.
3. FIXED CHARGE is a part of the utility that a consumer is required to pay even he does not use that utility.
4. VARIABLE CHARGE is a charge that a consumer is required to pay as per his usage.
5. SURCHARGE is an additional fee imposed on the consumer in addition to the standard basic rates of the utility.
6. ELECRTRICITY BILL is a statement giving details of the amount of electricity used by a consumer over a specific period of time. It shows the number of units used, the tariff, the fixed charge, surcharges and energy tax.

ALGORITHM FOR CALCULATING ELECTRICITY BILL	
Step 1	Find the number of units consumed in that month.
Step 2	Calculate the energy charges as per the tariff plan.
Step 3	Calculate the fixed charge according to the connection load.
Step 4	Calculate the surcharge as declared.
Step 5	Calculate the energy tax as per the government policy
Step 6	Add up Energy Charges, Fixed Charge, Surcharge and Energy Tax to get the final Electricity Bill

7. WATER BILL is a regular charge for the use of a local water supply. It consists on a charge based on the amount of water used by a consumer over a specific period of time and a set of charges such as meter rent, service charge, water cess, etc.

ALGORITHM FOR CALCULATING WATER BILL	
Step 1	Find the number of units consumed in that month.
Step 2	Calculate the water charges as per the tariff plan.
Step 3	Calculate the service charge as given.
Step 4	Note the meter rent.
Step 5	Add up Water Charges, Service Charge and to get the final Electricity Bill
Step 6	Add up Energy Charges, Fixed Charge, Surcharge and Meter Rent to get the final Water Bill.

BANKING

1. BANKING is a financial system that involves services offered by financial institutions to customers. It involves accepting deposits, lending money, facilitating transactions, etc.
2.

SB is savings bank, RD is recurring deposit, P is the principal amount, r is the rate of interest, n is the number of months		
Interest on SB Account = $\mathbf{P} \times \frac{R}{1200}$	Interest on a RD Account = $\frac{P \times n(n+1) \times R}{2400}$	M V of a RD Account = $\mathbf{P(n)} + \frac{Pn(n+1) \times R}{2400}$

SHARES & DIVIDEND

1	Market Price = Original Price + Premium or Original Price – Discount		
2	Investment = No. of shares x Market Price or No. of shares x Original Price		
3	Annual Income = No. of shares x Original Price x $\frac{\text{Dividend}}{100}$		
4	Percentage Return = $\frac{\text{Annual Income}}{\text{Investment}}$ x 100	5	Market Price x Yield % = Original Price x Dividend %

MENSURATION

2 D FIGURES					
	FIGURE	AREA	PERIMETER	DIAGONAL	NOMENCLATURE
1	Square	a^2	4 x a	$\sqrt{2}$ a	a = length of 1 side
2	Rectangle	L x B	2 (L + B)	$\sqrt{L^2 + B^2}$	L = length, B = Breadth
3	Parallelogram	Base x Height	2 (a + b)		a and b are lengths of parallel sides
4	Rhombus	$\frac{1}{2}(d_1 d_2)$	4 a		d_1 and d_2 are diagonals, a is length of 1 side
5	Trapezium	$\frac{1}{2}$h (a + b)	Sum of all sides		a, b are lengths of parallel sides, h is height
6	Triangle	$\frac{1}{2}$ x base x height	a + b + c		a, b, c are lengths of the sides of the triangle
7	Equilateral Triangle	$\frac{\sqrt{3}}{4}a^2$	3 a	Height = $\frac{\sqrt{3}}{2}$ a	a is the length of each side
8	Isosceles Triangle	$\frac{b}{4} \times \sqrt{4a^2 - b^2}$	2a + b	h = $\frac{1}{2} \times \sqrt{4a^2 - b^2}$	a = equal side, b = base, h = height
9	Circle	πr^2	$2\pi r$		r = radius
10	Semi – circle	$\frac{1}{2}\pi r^2$	$\pi r + 2r$		r = radius
11	Ring or Track	$\pi (R^2 - r^2)$	$2\pi (R + r)$	Width = R – r	R = external radius, r = internal radius
12	Sector	$\pi r^2 \times \frac{\theta}{360°}$	$2\pi r \frac{\theta}{360°} + 2r$		θ = angle between the radii

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3 D FIGURES

	FIGURE	CSA/ LSA	TSA	VOLUME	NOMENCLATURE
1	Cube	$4a^2$	$6a^2$	a^3	a = length of 1 side
2	Cuboid	$2h(l + b)$	$2(lh + bh + lb)$	$l \times b \times h$	l = length, b = breadth, h = height
3	Cylinder	$2\pi rh$	$2\pi r(h + r)$	$\pi r^2 h$	r = radius, h = height
4	Hollow Cylinder	$2\pi h(R + r)$	$\pi(R + r)(2h + R - r)$	$2\pi h(R^2 + r^2)$	R = ext. radius, r = int. radius, h = height
5	Cone	πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$	r = radius, h = height, l = slant height
6	Sphere	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r = radius
7	Hemi- sphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$	r = radius
8	Frustum	$\pi l(R + r)$	$\pi[l(R + r) + R^2 + r^2]$	$\frac{\pi h}{2}(R^2 + r^2 + Rr)$	l = slant height, h = perpendicular height
9	Pyramid	$\frac{1}{2} \times P \times l$	$(\frac{1}{2} \times P \times l) + B$	$\frac{1}{2} \times B \times h$	P = perimeter of base, l = slant height, B = base area, h = perpendicular height
10	Prism	$P \times h$	$(P \times h) + 2B$	$B \times h$	P = perimeter of base, B = base area h = perpendicular height,

ALGEBRA

ALGEBRAIC IDENTITIES

1	$(a + b)^2 = a^2 + 2ab + b^2$	**11**	$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$	**21**	$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
2	$(a - b)^2 = a^2 - 2ab + b^2$	**12**	$(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ac$	**22**	$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
3	$(a + \frac{1}{a})^2 = a^2 + 2 + \frac{1}{a^2}$	**13**	$(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ac$	**23**	$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
4	$(a - \frac{1}{a})^2 = a^2 - 2 + \frac{1}{a^2}$	**14**	$(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ac$	**24**	$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
5	$a^2 - b^2 = (a + b)(a - b)$	**15**	$a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ac)$	**25**	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
6	$a^2 + b^2 = (a + b)^2 - 2ab$	**16**	$(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$	**26**	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
7	$a^2 + b^2 = (a - b)^2 + 2ab$	**17**	$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$	**27**	$4ab = (a + b)^2 - (a - b)^2$
8	$(a + b)^2 = (a - b)^2 + 4ab$	**18**	$a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$	**28**	$a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2$
9	$(a - b)^2 = (a + b)^2 - 4ab$	**19**	$(a + b + c)(ab + bc + ca) = (b + c)(c + a)(a + b) + abc$	**29**	$a^4 - b^4 = (a + b)(a - b)(a^2 + b^2)$
10	$2(a^2 + b^2) = (a + b)^2 + (a - b)^2$	**20**		**30**	

SURDS & INDICES

1. SURD is an irrational number. It can be of the form $\sqrt{x}$, $\sqrt[3]{x}$, …, $\sqrt[n]{x}$, where x is any rational number and n is a positive integer.
2. LIKE SURDS are those 2 or more surds whose ratio is rational. [$\sqrt{12}$ and $\sqrt{27}$, $3\sqrt[3]{4}$ and $-2\sqrt[3]{4}$, …]
3. PURE SURD is that surd w2hich consists of a single term like $\sqrt{7}$, $\sqrt{15}$, $\sqrt[3]{4}$, etc.
4. MIXED SURD is a product of a rational and an irrational number like $3\sqrt{2}$, $\sqrt{12}$, etc.
5. BINOMIAL QUADRATIC SURD is of the form $a\sqrt{b} \pm c\sqrt{d}$
6. CONJUGATE SURDS are 2 mixed quadratic surds of the form $a + \sqrt{b}$ and $a - \sqrt{b}$

NOTE
1. In order to find the conjugate keep the rational part as it is and change the sign of the irrational part.
2. Product of a binomial quadratic surd and its conjugate is always rational.
3. If the sum and product of 2 mixed quadratic surds is rational then they are conjugate surds.
4. If the difference and product of both $a + \sqrt{b}$ and $c + \sqrt{d}$ is rational then the condition is $a = -c$.

PROPERTIES OF INDICES

1	$\sqrt[n]{a} = a^{\frac{1}{n}}$	5	$a^m \times a^n = a^{m+n}$	9	$a^0 = 1$
2	$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ or $(ab)^{\frac{1}{n}}$	6	$a^m \div a^n = a^{m-n}$	10	$a^1 = a$
3	$\sqrt[n]{a} \div \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$ or $\left(\frac{a}{b}\right)^{\frac{1}{n}}$	7	$(a^m)^n = a^{mn}$	11	$a^m \div b^m = \left(\frac{a}{b}\right)^m$
4	$(\sqrt[n]{a})^m = a^{\frac{m}{n}}$	8	$a^m \times b^m = (ab)^m$	12	$a^{-m} = \frac{1}{a^m}$

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LOGARITHM

1	**Product Law** : $\log_m (ab) = \log_m a + \log_m b$				3	**Power Law** : $\log_m a^n = n \log_m a$			
2	**Quotient Law** : $\log_m \left(\frac{a}{b}\right) = \log_m a - \log_m b$				4	**Change of Base Law** : $\log_b a = \frac{\log_m a}{\log_m b}$			
5	$\log_m 1 = 0$	6	$\log_m m = 1$	7	$\log_e 0 = -\infty$	8	$\log_e e^x = x$	9	$e^{\log_e x} = x$
10	$\log_b a = \frac{1}{\log_a b}$	11	$a^b = e^{b \log a}$	12	$\log_m \frac{1}{x} = -\log_m x$	13	$\log_{a^n} x^m = \frac{m}{n} \log_a x$	14	$\log_{a^n} x = \frac{1}{n} \log_a x$

1. CHARACTERISTIC is the integral part of a logarithm or the number to the left of the decimal. It is always positive for any number greater than 1.
2. MANTISSA is the decimal part of the logarithm. It is always positive and is calculated by taking the first 4 digits of the decimal part starting from the first non – zero digit.

ALGORITHM TO FIND THE LOG AND ANTILOG OF A GIVEN NUMBER	
Step 1	Write down the characteristic. The characteristic of the logarithm of any number greater than 1 is always positive and should be 1 less than the number of digits in the integral part. If the number is less than 1 then the characteristic is negative and 1 more than the number of zeroes to the right of the decimal.
Step 2	Find the mantissa by using the log table. From the decimal part of the given number take the first 4 digits that come immediately after the decimal and /or after any zeroes in the decimal part. Use the first 2 digits to locate the row in the log table. Then write the number of that row corresponding to the column shown by the third digit. Add the mean difference using the fourth digit.
Step 3	Perform the required operation. The mantissa should always be a positive quantity. If it is negative convert it into a positive one. For example $-3.1541 = -4 + (1 - 0.1541) = \bar{4}.8459$
Step 4	To find the anti – logarithm of a number use the antilog table. The first digits of the mantissa work as the row number, the third digit as the column number and the fourth digit as the mean difference. Add the values so obtained.
Step 5	To the characteristic add 1 and insert the decimal point after that many digits from the left.

SIMULTANEOUS LINEAR EQUATIONS IN TWO VARIABLES

1. SUBSTITUTION METHOD

ALGORITHM FOR SUBSTITUTION METHOD	
Step 1	Write the two equations as $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Step 2	Choose any one equation and find the value of one variable (say y) in terms of the other (say x)
Step 3	Substitute the value of y obtained above in the other equation to get an equation in one variable, x.
Step 4	Solve the equation obtained in step 3 to get the value of x.
Step 5	Substitute this value of x in the equation obtained in step 2 to get the value of y.

2. ELIMINATION METHOD

ALGORITHM FOR ELIMINATION METHOD	
Step 1	Write the two equations as $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Step 2	Multiply both equations by a certain number so as to make the coefficients of the variable to be eliminated equal.
Step 3	Add or subtract the equations obtained in step 2. Add if the equal coefficients have opposite signs else subtract.
Step 4	Solve the equation obtained in step 3 to get the value of x.
Step 5	Substitute this value of x in the any one equation to get the value of y.

3. CROSS MULTIPLICATION METHOD

ALGORITHM FOR CROSS MULTIPLICATION METHOD	
Step 1	Write the two equations as $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Step 2	Write $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$
Step 3	Obtain the value of x by equating the 1st and 3rd expressions and that of y by equating the 2nd and 3rd expressions.

4. CONSISTENT system of linear equations in two variables is that which has at least one solution.
5. INCONSISTENT system of linear equations in two variables is that which has no solution.
6. UNIQUE SOLUTION is when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ [The 2 straight lines are intersecting at one point]
7. NO SOLUTION is when $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ [The 2 straight lines are parallel]
8. INFINITE SOLUTION is when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ [The 2 straight lines are coincident]

FORMULAE SHEET

LINEAR INEQUATIONS

1. LINEAR INEQUATION is a statement involving one or more variables and the sign of inequality like <, >, ≤ or ≥

TYPES OF LINEAR INEQUATIONS			
In 1 variable	In 2 variables	Quadratic	Rational
$ax + b < 0$ or $ax + b > 0$ or $ax + b \le 0$ or $ax + b \ge 0$	$ax + by < c$ or $ax + by > c$ or $ax + by \le c$ or $ax + by \ge c$	$ax^2 + bx + c < 0$ or $ax^2 + bx + c > 0$ or $ax^2 + bx + c \le 0$ or $ax^2 + bx + c \ge 0$	$\frac{ax+b}{cx+d} < k$ or $\frac{ax+b}{cx+d} > k$ or $\frac{ax+b}{cx+d} \le k$ or $\frac{ax+b}{cx+d} \ge k$

2. SOLUTION SET is the set of all possible solutions of an inequation.

RULES FOR SOLVING LINEAR INEQUATION	
Rule 1	The sign of inequality does not change when the same number is added to or subtracted from both sides of the inequation.
Rule 2	The sign of inequality does not change when both sides of the inequation are multiplied or divided by the same positive number.
Rule 3	The sign of inequality changes when both sides of the inequation are multiplied or divided by the same negative number.
Rule 4	The sign of inequality changes when transposition takes place in the inequation.

ALGORITHM FOR SOLVING A LINEAR INEQUATION	
Step 1	Take all terms containing the variable on one side of the inequation and the constant terms on the other side.
Step 2	Simplify both sides of inequality in their simplest form to reduce the inequation in the form $ax \le b$ or $ax \ge b$ or $ax < b$ or $ax > b$
Step 3	Solve the inequation in step 2 by dividing both sides of the inequation with the coefficient of the variable.
Step 4	Write the solution set obtained in step 3 in the form of an interval on the real line.

3. GRAPHING LINEAR INEQUATION ON A NUMBER LINE :
   a) For an inequation of the type $a < x < b$, draw circles on a and b and join them with a straight line.
   b) For an inequation of the type $a \le x \le b$, draw and darken the circles at a and b and join them with a straight line.
   c) For an inequation of the type $x < a$ or $x > a$, draw a circle at a and draw a line to the left or right as needed.
   d) For an inequation of the type $x \le a$ or $x \ge a$, draw and darken the circle at a and draw a line to the left or right as needed.

ALGORITHM FOR SOLVING A RATIONAL LINEAR INEQUATION	
Step 1	Take all terms of the inequation $\frac{ax+b}{cx+d} < k$ or $\frac{ax+b}{cx+d} > k$ or $\frac{ax+b}{cx+d} \le k$ or $\frac{ax+b}{cx+d} \ge k$ to LHS.
Step 2	Simplify LHS of the inequation to obtain $\frac{px+q}{rx+s} < 0$ or $\frac{px+q}{rx+s} > 0$ or $\frac{px+q}{rx+s} \le 0$ or $\frac{px+q}{rx+s} \ge 0$
Step 3	Make the coefficient of x positive in both numerator and denominator.
Step 4	Equate numerator and denominator separately to zero and obtain the values of x.
Step 5	Plot the critical points obtained in step 4 on real line. These points will divide the real line into three regions.
Step 6	The right most region will be positive and in other regions it will alternatively negative and positive,
Step 7	Select appropriate region on the basis of the sign of inequation obtained in step 4.
Step 8	Write these regions in the form of intervals to obtain the desired solution sets of the given inequation.

IMPORTANT RESULTS	
Result 1	If a is a positive real number then for $\|x\| < a \Rightarrow -a < x < a$ and for $\|x\| \le a \Rightarrow -a \le x \le a$
Result 2	If a is a positive real number then for $\|x\| > a \Rightarrow x < -a$ or $x > a$ and for $\|x\| \ge a \Rightarrow x \le -a$ or $x \ge a$.
Result 3	If r is a positive real number and a is fixed number then for $\|x - a\| < r \Rightarrow a - r < x < a + r$ and for $\|x - a\| \le r \Rightarrow a - r \le x \le a + r$
Result 4	If r is a positive real number and a is fixed number then for $\|x - a\| > r \Rightarrow x < a - r$ or $x > a + r$ and for $\|x - a\| \ge r \Rightarrow x \le a - r$ or $x \ge a + r$

POLYNOMIALS

1. POLYNOMIAL is a mathematical expression consisting of variables, constants, coefficients and exponents.
2. DEGREE of a polynomial is the highest power of its variable.

Linear Polynomial		Quadratic Polynomial				Cubic Polynomial				
Degree	Zeroes	Degree	Zeroes	$\alpha + \beta$	$\alpha\beta$	Degree	Zeroes	$\alpha + \beta + \gamma$	$\alpha\beta + \beta\gamma + \gamma\alpha$	$\alpha\beta\gamma$
1	1	2	2	$\frac{-b}{a}$	$\frac{c}{a}$	3	3	$\frac{-b}{a}$	$\frac{c}{a}$	$\frac{-d}{a}$
α, β and γ are the zeroes of the polynomial. Quadratic polynomial will have only two zeroes, while a cubic polynomial will have three zeroes.										

FORMULAE SHEET

QUADRATIC EQUATIONS

1. GENERAL FORM of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are real numbers.
2. QUADRATIC EQUATION can be solved by **FACTORIZATION**, by using the **FORMULA** and by **COMPLETING** the square.
3. ROOTS of a quadratic equation are those values of the variable which satisfy the given equation,
4. FACTORIZATION METHOD : Split the middle term that is bx such that their sum is equal to the product of the first (ax^2) and the last (c) terms.

Express the quadratic polynomial $ax^2 + bx + c$ as a product of linear factors like $(px + q)$ and $(rx + s)$
Then $ax^2 + bx + c = 0$ $\Rightarrow (px + q)(rx + s) = 0$ $\Rightarrow px + q = 0$ or $rx + s = 0$ $\Rightarrow x = \frac{-q}{p}$ or $x = \frac{-s}{r}$

NOTE
1. A quadratic equation cannot have more than two roots.
2. The set of all roots of an equation in a given domain is called the **SOLUTION SET** of the equation.

5. FORMULA METHOD : Use the formula $\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$, where $\mathbf{b^2 - 4ac}$ is called the DISCRIMINANT (D)

NATURE OF THE ROOTS			
1	If D > 0 and not a perfect square, than the roots are real and irrational.	3	If D = 0 than the roots are real and equal
2	If D > 0 and a perfect square, than the roots are real and rational	4	If D < 0 than the roots are imaginary.

NOTE
For quadratic equations with real coefficients complex roots always occur in conjugate pairs.

6. If α and β are the roots of a quadratic equation, than $\boldsymbol{\alpha = \frac{-b + \sqrt{D}}{2a}}$ and $\boldsymbol{\beta = \frac{-b - \sqrt{D}}{2a}}$
7. Sum of the roots $\boldsymbol{(\alpha + \beta) = -\frac{b}{a}}$
8. Product of the roots $\boldsymbol{(\alpha\beta) = \frac{c}{a}}$
9. COMPLETING THE SQUARE : Write $ax^2 + bx + c = 0$ as $p(x + m)^2 + q = 0$

ALGORITHM FOR COMPLETING THE SQUARE	
Step 1	Write the equation as $x^2 + bx + c = 0$. Coefficient of x^2 has to be 1. If not take it as the common factor.
Step 2	Take half of the coefficient of x, square it and add and subtract to get $x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$
Step 3	Write $(x + \frac{b}{2})^2 = \left(\frac{b}{2}\right)^2 - c$. Let $\left(\frac{b}{2}\right)^2 - c = p$
Step 4	Then $x = \pm\sqrt{p} - \frac{b}{2}$

10. If α and β are the roots of a quadratic equation then the equation is $\mathbf{x^2 - (\alpha + \beta)x + \alpha\beta = 0}$.
11. If α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then $\boldsymbol{ax^2 + bx + c = a(x - \alpha)(x - \beta)}$.
12. If the roots are equal in magnitude but opposite in signs than the coefficient of x is 0.
13. If one root of the quadratic equation $ax^2 + bx + c = 0$ is 1 then the other root is $\frac{c}{a}$.

COMPLEX NUMBERS

1	COMPLEX NUMBER is expressed in the form $z = a + ib$, where a and b are real numbers and $i = \sqrt{-1}$		
2	REAL PART (Re) of $z = a + ib$ is a.	**3**	IMAGINARY PART (Im) of $z = a + ib$ is ib
4	CONJUGATE of a complex number is $\bar{z} = a - ib$	**5**	MODULUS of a complex number $\lvert z\rvert = \sqrt{a^2 + b^2}$

6	$\mathbf{i = \sqrt{-1}}$	**7**	$\mathbf{i^2 = -1}$	**8**	$\mathbf{i^3 = -i}$	**9**	$\mathbf{i^4 = 1}$

1. ADDITION OF COMPLEX NUMBERS : If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ then $\mathbf{z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)}$.
2. SUBTRACTION OF COMPLEX NUMBERS : If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ then $\mathbf{z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)}$.
3. MULTIPLICATION OF COMPLEX NUMBERS : If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ then $\mathbf{z_1 z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1)}$
4. DIVISION OF COMPLEX NUMBERS : If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ then $\mathbf{\frac{z_1}{z_2} = \left\{\frac{a_1 a_2 + b_1 b_2}{(a_2)^2 + (b_2)^2}\right\} + i\left\{\frac{a_1 a_2 - b_1 b_2}{(a_2)^2 + (b_2)^2}\right\}}$
5. RECIPROCAL OF A COMPLEX NUMBER : If $z = a + ib$ then $\mathbf{\frac{1}{z} = \frac{a - ib}{a^2 + b^2}}$
6. SQUARE ROOT OF A COMPLEX NUMBER : If $z = a + ib$ then $\mathbf{\sqrt{z} = \pm\left[\sqrt{\frac{1}{2}\left\{\sqrt{a^2 + b^2} + a\right\}}\right] + i\left[\sqrt{\frac{1}{2}\left\{\sqrt{a^2 + b^2} - a\right\}}\right]}$

ALGORITHM TO FIND THE SQUARE ROOT OF A COMPLEX NUMBER	
Step 1	Take $\sqrt{a + ib}$ = x + iy [z = $a + ib$]
Step 2	Square both sides of the equation to get $a + ib = (x + iy)^2 \Rightarrow a + ib = x^2 - y^2 + 2ixy$ [$i^2 = -1$]
Step 3	Equating the real and imaginary parts to get $a = x^2 - y^2$ and $b = 2xy$
Step 4	Now $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2 \Rightarrow (x^2 + y^2)^2 = a^2 + b^2$ [$x^2 - y^2 = a$ and $4x^2y^2 = (2xy)^2 = b^2$.
Step 5	Taking square root on both sides obtain $x^2 + y^2 = \sqrt{a^2 + b^2}$ [Remember $x^2 + y^2 \geq 0$]
Step 6	Solve the equations to get $x = \pm\left[\sqrt{\frac{1}{2}\left\{\sqrt{a^2 + b^2} + a\right\}}\right]$ and $y = \pm\left[\sqrt{\frac{1}{2}\left\{\sqrt{a^2 + b^2} - a\right\}}\right]$ [$x^2 + y^2 = \sqrt{a^2 + b^2}$ and $x^2 - y^2 = a$]
Step 7	If b is positive $\sqrt{a + ib} = \pm\left[\sqrt{\frac{1}{2}\left\{\sqrt{a^2 + b^2} + a\right\}} + i\sqrt{\frac{1}{2}\left\{\sqrt{a^2 + b^2} - a\right\}}\right]$
Step 8	If b is negative $\sqrt{a + ib} = \pm\left[\sqrt{\frac{1}{2}\left\{\sqrt{a^2 + b^2} + a\right\}} - i\sqrt{\frac{1}{2}\left\{\sqrt{a^2 + b^2} - a\right\}}\right]$

7. POLAR FORM of a complex number z = $x + iy$ is **z = r (cos θ + *i* sin θ)**, where $r = \sqrt{x^2 + y^2}$ and $\tan\alpha = \left|\frac{y}{x}\right|$

ALGORITHM TO WRITE A COMPLEX NUMBER IN THE POLAR FORM	
Step 1	Take the complex number $z = x + iy$
Step 2	Find $r = \lvert z \rvert = \sqrt{x^2 + y^2}$
Step 3	Find the angle α by using $\tan\alpha = \left\lvert\frac{y}{x}\right\rvert$
Step 4	Take $\theta = (\pi - \alpha)$
Step 5	Then the polar form of $z = x + iy$ will be $z = r(\cos\theta + i\sin\theta)$

NOTE	1. In $z = x + iy$ if $x > 0$ and $y > 0$, then $\theta = \alpha$ 2. In $z = x + iy$ if $x < 0$ and $y > 0$, then $\theta = \pi - \alpha$ 3. In $z = x + iy$ if $x < 0$ and $y < 0$, then $\theta = -(\pi - \alpha)$ 4. In $z = x + iy$ if $x > 0$ and $y < 0$, then $\theta = -\alpha$

8. ARGAND PLANE is a plane used to represent a complex number of the form $z = x + iy$. The modulus of the complex number represents the distance of the point (x, y) from the origin.
9. ARGUMENT or AMPLITUDE of a complex number is the angle **θ** between the positive real axis and the line joining the origin and z. It is written as ***arg (z)*** or ***amp (z)***.
10. POLAR COORDINATES are an ordered pair of real numbers **(*r*, θ)** that uniquely determines the position of a point P on a plane formed by taking the positive direction of the x – axis as the initial line and the origin as the pole.

SETS

1. SET is a well – defined collection of objects.

FORMS OF SETS	
ROOSTER FORM	In this form a set is described by listing elements separated by commas and within curved brackets { }. **Example : A = {m, a, t, h, e, i, c, s}**
SET BUILDER FORM	In this form the set is described within curved brackets { } by using a variable and a rule to specify the properties of its elements. **Example : A = {*x* : *x* is a letter of the word MATHEMATICS}**
STATEMENT FORM	In this form the set is described in a well – defined manner enclosed in curved brackets { }. **Example : A = {letters in the word mathematics}**

2. EMPTY SET is a set that has no elements at all.
3. SINGLETON SET is a set that consists of only 1 element.
4. FINITE SET is a set that consists of a finite number of elements.
5. CARDINAL NUMBER OF A FINITE SET is the number of distinct elements contained in the set.
6. INFINITE SET is a set that consists of infinite number of elements.
7. EQUAL SETS are those two sets whose each and every element is the same.
8. EQUIVALENT SETS are those two sets in which the number of elements is the same but the elements are different.
9. SUBSET : If A and B are 2 sets such that every element of A is in B, then A is a subset of B. $\mathbf{A \subseteq B}$
10. PROPER SUBSET : If A is a subset of B and A ≠ B, then A is a proper subset of B. $\mathbf{A \subset B}$
11. POWER SET is the set formed by all the subsets of a given set A. It is denoted by ***P (A)***.
12. NUMBER OF SUBSETS = 2^n where n is the number of elements in the set.
13. NUMBER OF PROPER SUBSETS = $2^n - 1$ where n is the number of elements in the set.

14. UNIVERSAL SET is that set of which all other sets in consideration are subsets.
15. UNION OF SETS : The union of 2 sets A and B is the set of all those elements which are either in A or in B or in both. $\mathbf{A \cup B}$
16. INTERSECTION OF SETS : The intersection of 2 sets A and B is the set of all those elements which are common to both A and B. $\mathbf{A \cap B}$.
17. DISJOINT SETS : Two sets are said to be disjoint if they have no common element. $A \cap B = \phi$
18. COMPLEMENT of a set are all those elements which are in the universal set but not in the set itself.

SOME RESULTS ON SETS

1	$A \cup \phi = A$	6	$n(A - B) = n(A) - n(A \cap B)$	11	$A \times (B \cap C) = (A \times B) \cap (A \times C)$
2	$A \cap \phi = \phi$	7	$n(B - A) = n(B) - n(A \cap B)$	12	$A \times (B - C) = (A \times B) - (A \times C)$
3	$A \subset B$, then $A \cup B = B$	8	$(A \cup B)' = A' \cap B'$	13	$n(A \times B) = n(A) \times n(B)$
4	$A \subset B$, then $A \cap B = A$	9	$(A \cap B)' = A' \cup B'$	14	$A - (B \cup C)' = (A - B) \cup (A - C)$
5	$n(A \cup B) = n(A) + n(B) - n(A \cap B)$	10	$A \times (B \cup C) = (A \times B) \cup (A \times C)$	15	$A - (B \cap C)' = (A - B) \cap (A - C)$

DE MORGAN'S LAW

1ST LAW	The complement of the union of two sets is equal to the intersection of the complement of each set.	$(A \cup B)' = (A)' \cap (B)'$
2ND LAW	The complement of the intersection of two sets is equal to the union of the complement of each set.	$(A \cap B)' = (A)' \cup (B)'$

TYPES OF INTERVALS

NOTATION	TYPE	SET DESCRIPTION	NOTATION	TYPE	SET DESCRIPTION
[a, b]	Closed Interval	For $x \in R : a \le x \le b$	(a, ∞)	Infinite open	For $x \in R : x > a$
(a, b)	Open Interval	For $x \in R : a < x < b$	$[a, \infty)$	Infinite closed	For $x \in R : x \ge a$
[a, b)	Right half open Interval	For $x \in R : a \le x < b$	$(-\infty, a]$	Infinite closed	For $x \in R : x < a$
(a, b]	Left half open Interval	For $x \in R : a < x \le b$	$(-\infty, \infty)$	Set of all real numbers	For $x \in R : x \le a$

RELATIONS

1. ORDERED PAIR is that which consists of two objects or elements whose coordinates occur in a given fixed order.
2. CARTESIAN PRODUCT OF SETS : If A and B are two non – empty sets then the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the Cartesian product of the sets A and B and is denoted by A x B (read as "A cross B").
3. NUMBER OF ELEMENTS IN A x B = $n(A) \times n(B)$
4. RELATION is a set of ordered pairs that is a subset of A x B.
5. TOTAL NUMBER OF RELATIONS : If set A has $\boldsymbol{m}$ elements and set B has $\boldsymbol{n}$ elements than the total number of relations from A to B is 2^{mn}.
6. REFLEXIVE RELATION is a binary relation on a set where every element of the set is related to itself. It is a subset of A x A such $(a, a) \in R$ for each and every $a \in A$. It will not be reflexive if there is at least one element $a \in A$ such that $(a, a) \notin R$.
7. SYMMETRIC RELATION is a binary relation on a set where if one element is related to another then the other element is also related to the first. It is a subset of A x A such $(a, b) \in R$ then $(b, a) \in R$, for each and every $a, b \in A$.
8. ANTI – SYMMETRIC RELATION is a relation on a set where no pair of distinct elements are related to each other.
9. TRANSITIVE RELATION is a binary relation on a set where if the first element is related to the second and the second is related to the third then the first element is also related to the third. It is a subset of A x A such that if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$ for each and every $a, b, c \in A$.
10. EQUIVALENCE RELATION is a relation on a set that is reflexive, symmetric and transitive for everything in the set.
11. PARTIAL ORDER RELATION is a relation on a set that is reflexive, transitive and anti – symmetric at the same time.
12. VOID RELATION is a relation on a set where no element of the set is related to any other element.
13. UNIVERSAL RELATION is a relation on a set where every element of the set is related to each other.
14. IDENTITY RELATION is a relation on a set where every element of the set is related to itself only.

IMPORTANT RESULTS

1	If either A or B is a null set then A x B will also be a null set. $[A = \{m, n\}$ and $B = \phi$ then $A \times B = \phi]$
2	If either A or B is an infinite set then A x B will also be an infinite set.
3	If A and B are two non – empty sets having n elements then A x B or B x A will have n^2 elements in common.
4	If A = B then $A \times A = A^2$
5	Domain of R is all the first elements inside the parenthesis { }.
6	Range of R is all the second elements inside the parenthesis { }.

FUNCTIONS

1. ONE – ONE FUNCTION (INJECTION) is a function from A $\rightarrow$ B such that all the elements in A have a different image in B.
2. MANY – ONE FUNCTION is a function from A $\rightarrow$ B such that 2 or more elements from set A have the same image in B.

ALGORITHM TO CHECK IF THE FUNCTION IS		
	ONE – ONE OR NOT	MANY – ONE OR NOT
Step 1	Take 2 arbitrary elements a_1 and a_2 in the domain of f	Take 2 arbitrary elements $a_1, a_2 \in A$
Step 2	Put $f(a_1) = f(a_2)$ and solve the equation	Put $f(a_1) = f(a_2)$ and solve the equation
Step 3	If it gives $a_1 = a_2$, only then it is a one – one function	If it gives $a_1 \neq a_2$, then it is a many – one function

3. ONTO FUNCTION (SURJECTION) is a function from A $\rightarrow$ B such that each element of B is a pre – image of an element from set A.
4. INTO FUNCTION is a function from A $\rightarrow$ B such that there exists an element in B having no pre – image in A.
5. REAL FUNCTION is a function that maps each element of its domain to a real number. It is normally represented by letters like f, g or h. For example A = f (x). Domain of the function is all real numbers. The range of a real function lies within the real numbers.
6. CONSTANT FUNCTION is that function which has the same output value irrespective of the input value. It is normally represented as $f(x) = k$ for all $x \in R$. The domain is a set of all real numbers while the range is a singleton set.
7. IDENTITY FUNCTION is a function which returns the same value which was used as its argument. It is represented as $f(x) = x$ for all values of x. Domain is R and the Range is also R
8. MODULUS FUNCTION is that function which returns the absolute value of a number. Irrespective of the input the output is always a positive value. It is represented as $f(x) = |x|$. The domain of a modulus function is all real numbers while the range is all positive real numbers including 0.
9. RECIPROCAL FUNCTION is that function which is the inverse of another function. The reciprocal function of $f(x) = x$ is $f(x) = \frac{1}{x}$. The reciprocal function is not defined when x = 0. Domain of this function is R – {0}. Range is also R – {0}
10. SIGNUM FUNCTION is that function which gives the sign for the given values of x. For x value greater than 0, the value of the output is + 1. For x value lesser than 0, the value of the output is – 1 and for x value equal to 0, the output is equal to 0. Domain of this function is R and the Range is {– 1, 0, 1}
11. SQUARE FUNCTION is a real function that is defined by the squaring operation. It is represented as $f(x) = x^2$. Domain of this function is all real numbers and the range is all positive real numbers including 0.
12. SQUARE ROOT FUNCTION is a mathematical function that maps a set of non – negative real numbers to itself. It is represented by the equation $f(x) = \sqrt{x}$. Domain is a set of all non – negative real numbers including 0. $[0, \infty[$. Range of square root function is also a set of all non – negative real numbers greater than or equal to 0. $[0, \infty[$.
13. GREATEST INTEGER FUNCTION (STEP FUNCTION) is that function which takes a real number as input and the output is the largest integer that is less than or equal to that number. It is represented by the equation $f(x) = \lfloor x \rfloor$. Domain of this function is R. Range is Z.
14. SMALLEST INTEGER FUNCTION (CEILING FUNCTION) is that function which takes a real number as input and the output is the smallest integer that is greater than or equal to that number. It is represented by the equation $f(x) = \lceil x \rceil$. Domain of this function is R. Range is Z.
15. POLYNOMIAL FUNCTION is that function which uses only positive integer exponents of a variable in an equation. It is of the form $f(x) = a\,x^n + a_1\,x^{n-1} + a_2\,x^{n-2} + \ldots + a_n$. The domain and range of a polynomial function depends on the degree of the polynomial.
16. TRANSCENDENTAL FUNCTION is that which is not algebraic. It cannot be produced by algebraic operations like addition, subtraction, multiplication and inverse operations.
17. EXPONENTIAL FUNCTION is a mathematical function in the form $f(x) = a^x$ where x is a variable and a is a constant. Domain is the set of all real numbers. Range is greater than domain if a > 0 and is less than the domain if a < 0.
18. LOGARITHMIC FUNCTION is the inverse of an exponential function. The base in a logarithmic function and in an exponential function are the same. The logarithmic function is represented as $f(x) = \log_a x$, where a > 0 and a ≠ 1. Here the domain is a set of all positive real numbers from 0 to ∞ and range is the set R of all real numbers.
19. RATIONAL FUNCTION is any function that can be defined by a rational fraction such that both the numerator and the denominator are polynomials. It is represented as $f(x) = \frac{p(x)}{q(x)}$. The domain of a rational function is the set of values which the independent variable is allowed to assume. Range is the set of values which the dependent variable is allowed to assume.
20. ODD FUNCTION is that where $f(-x) = -f(x)$ for every x in its domain.
21. EVEN FUNCTION is that where $f(-x) = f(x)$ for every x in its domain.

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	FUNCTION	DOMAIN	RANGE
1	$f(x) = ax + b, a \neq 0$	R	R
2	$f(x) = x^2$	R	0 to ∞
3	$f(x) = \frac{1}{x}$	R – {0}	R – {0}
4	$f(x) = \sqrt{a^2 - x^2}$, a > 0	$[-a, a]$	$[0, a]$
5	$f(x) = \sqrt{x^2 - a^2}$, a > 0	$(-\infty, -a] \cup [0, \infty]$	$[0, \infty]$
6	$f(x) = \frac{1}{\sqrt{a^2 - x^2}}$, a > 0	$[0, a]$	$[\frac{1}{a}, \infty]$
7	$f(x) = \frac{1}{\sqrt{x^2 - a^2}}$, a > 0	$(-\infty, -a] \cup [a, \infty]$	$[0, \infty]$
8	$f(x) = \lfloor x \rfloor$	R	I
9	$f(x) = \lceil x \rceil$	R	I
10	$f(x) = \lvert x \rvert$	R	$[0, \infty]$
11	$f(x) = e^x$	R	$[0, \infty]$
12	$f(x) = \log_a x$, a > 0	$[0, \infty]$	R
13	f (x) = Sin x	R	$[-1, 1]$
14	f (x) = Cos x	R	$[-1, 1]$
15	f (x) = Tan x	$R - (2n+1)\frac{\pi}{2}, n \in I$	R
16	f (x) = Cosec x	$R - (n\pi), n \in I$	$(-\infty, -1) \cup (1, \infty)$
17	f (x) = Sec x	$R - (2n+1)\frac{\pi}{2}, n \in I$	$(-\infty, -1) \cup (1, \infty)$
18	f (x) = Cot x	$R - (n\pi), n \in I$	R
19	$f(x) = \sin^{-1} x$	$[-1, 1]$	$[\frac{-\pi}{2}, \frac{\pi}{2}]$
20	$f(x) = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
21	$f(x) = \tan^{-1} x$	R	$[\frac{-\pi}{2}, \frac{\pi}{2}]$
22	$f(x) = \operatorname{cosec}^{-1} x$	$(-\infty, -1) \cup (1, \infty)$	$[\frac{-\pi}{2}, \frac{\pi}{2}] - 0$
23	$f(x) = \sec^{-1} x$	$(-\infty, -1) \cup (1, \infty)$	$(0, \pi) - \frac{\pi}{2}$
24	$f(x) = \cot^{-1} x$	R	$[0, \pi]$

SEQUENCES & SERIES

1. SEQUENCE is a list of numbers written in a particular order and in which repetition is allowed. It can be finite or infinite.
2. SERIES is the sum of a list of numbers that are generated according to a rule or pattern.
3. PROGRESSION is a sequence in which its terms increases or decreases
4. ARITHMETIC PROGRESSION (AP) is a sequence in which the difference between any two consecutive terms is always the same.
5. GEOMETRIC PROGRESSION (GP) is a sequence in which the ratio between any two consecutive terms is always the same.
6. HARMONIC PROGRESSION (HP) is a sequence of non – zero numbers such that the reciprocals of its terms is in arithmetic progression.

ARITHMETIC PROGRESSION		GEOMETRIC PROGRESSION		HARMONIC PROGRESSION	
1	$T_n = a + (n-1)d$	1	$Tn = ar^{n-1}$	1	$Tn = \frac{1}{a + (n-1)d}$
2	$T_n = S_n - S_{n-1}$	2	$S_n = a\frac{r^n - 1}{r - 1}$, for r > 1		
3	$S_n = \frac{n}{2}[2a + (n-1)d]$	3	$S_n = a\frac{1 - r^n}{1 - r}$, for r < 1		
4	$S_n = \frac{n}{2}(a + l)$	4	$S_{infinity} = \frac{a}{1 - r}$	2	$HM = \frac{2ab}{a + b}$
5	$AM = \frac{a + b}{2}$	5	$GM = \sqrt{ab}$		
NOMENCLATURE	(i) $\mathbf{T_n}$ is the n^{th} term (ii) **a** is the first term (iii) **d** is the common difference (iv) **r** is the common ratio (v) n is the number of terms (vi) l is the last term (vii) S_n is the sum of n terms.				

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BINOMIAL THEOREM

1. BINOMIAL EXPRESSION is an expression containing two terms.
2. The expansion of $(x + a)^n$ will contain $(n + 1)$ terms.
3. The expansion of $(x + a + b)^n$ will contain $\frac{(n+1)(n+2)}{2}$ terms
4. In the expansion of $(x + a)^n$ the degree of x will go on decreasing while that of a will go on increasing.
5. The sum of the degrees of x and a will always be equal to n.
6. If x and a are real numbers then for all $n \in N$, $(x + a)^n = x^n + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + \ldots + {}^nC_{n-1} x\, a^{n-1} + a^n$.
   For $(x - a)^n$ all even terms are negative.
7. GENERAL TERM of a Binomial Expansion is $T_{r+1} = {}^nC_r\ x^{n-r}\ a^r$ for $(x + a)^n$ or $T_{r+1} = (-1)^r\, x\ {}^nC_r\ x^{n-r}\ a^r$ for $(x - a)^n$.
8. MIDDLE TERM in a Binomial Expansion is $\left(\frac{n}{2} + 1\right)^{th}$ term if n is even and $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms if n is odd
9. GREATEST BINOMIAL COEFFICIENT is the binomial coefficient of the middle term of $(x + y)^n$.
10. If n is odd then $|(x + a)^n + (x - a)^n|$ and $|(x + a)^n - (x - a)^n|$ both have $\left(\frac{n+1}{2}\right)$ terms
11. If n is even then $|(x + a)^n + (x - a)^n|$ has $\left(\frac{n}{2} + 1\right)$ terms and $|(x + a)^n - (x - a)^n|$ has $\frac{n}{2}$ terms.
12. The term independent of x in the expansion of $\left(ax^p + \frac{b}{x^q}\right)^n$ is ${}^nC_r\, a^{n-r}\ b^r$ where $r = \frac{np}{p+q}$

IMPORTANT RESULTS			
1	In the expansion of $(x + y)^n$, r^{th} term from the end is equal to $(n - r + 2)^{th}$ term from the beginning.		
2	The expansions of $(x + y)^n$ and $(y + x)^n$ are equal but their respective terms are not equal.		
3	In the expansion of $\left(a + \frac{x}{b}\right)^n$, if the coefficients of x^r and x^{r-1} are equal then $\boldsymbol{n = r(ab + 1) - 1}$		
4	In the expansion of $\left(a + \frac{x}{b}\right)^n$, if the coefficients of x^r and x^{r+1} are equal then $\boldsymbol{n = ab(r + 1) + r}$		
5	If a, b, c are three consecutive coefficients in the expansion of $(1 + x)^n$ then $n = \frac{2ac + b(a+c)}{b^2 - ac}$ and $r = \frac{a(b+c)}{b^2 - ac}$		
If O is the sum of odd terms and E is the sum of even terms in the expansion of $(x + a)^n$, then			
(a)	$(x + a)^n = O + E$ and $(x - a)^n = O - E$.	**(b)**	$(x^2 - a^2)^n = O^2 - E^2$.
(c)	$(x + a)^{2n} + (x - a)^{2n} = 2(O^2 + E^2)$	**(d)**	$(x + a)^{2n} - (x - a)^{2n} = 4OE$

PASCAL'S TRIANGLE was invented by one of most the celebrated mathematician Blaise Pascal.
Each term in the table is derived by adding together the two terms in the line above which lie on either side of it.

MATHEMATICAL INDUCTION

1. MATHEMATICAL STATEMENT is a statement involving mathematical relations.
2. FIRST PRINCIPLE OF MATHEMATICAL INDUCTION states that if $P(n)$ is a statement involving a natural number n such that $P(n)$ is true for $n = 1$ and $P(k + 1)$ is true whenever $P(k)$ is true then $P(n)$ is true for all natural numbers.
3. SECOND PRINCIPLE OF MATHEMATICAL INDUCTION states that if $P(n)$ is a statement involving a natural number n such that $P(n)$ is true for $n = 1$ and $P(k + 1)$ is true whenever $P(n)$ is true for all n, where $1 \le n \le k$ then $P(n)$ is true for all natural numbers.

ALGORITHM FOR PMI	
Step 1	Obtain $P(n)$ and understand its meaning.
Step 2	Prove that the statement is true for $n = 1$.
Step 3	Assume that the statement is true for $n = k$, i.e. $P(k)$ is true.
Step 4	Now prove that $P(k + 1)$ is also true.
Step 5	Combining the results of Step 3 and Step 4, conclude by the first principle of mathematical induction that $P(n)$ is true for all $n \in N$.

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MATRICES

1. MATRIX is an arrangement of elements in rows and columns.
2. ORDER of a matrix is found by the number of rows and columns it has. A matrix having ***m* rows** and ***n* columns** is said to be of the order ***m* x *n*** (read as "m by n").
3. a_{ij} denotes an entry in a matrix that occurs in the i^{th} row and j^{th} column.
4. SQUARE MATRIX is that where the number of rows and number of columns are equal.
5. RECTANGULAR MATRIX is that where the number of rows and number of columns are not equal.
6. ROW MATRIX is a matrix having elements only 1 row.
7. COLUMN MATRIX is a matrix having elements only 1 column.
8. DIAGONAL MATRIX is a square matrix where all elements except those on the principal diagonal are 0.
9. IDENTITY MATRIX (UNIT MATRIX) is a diagonal matrix whose all elements are 1. It is denoted by I.
10. NULL MATRIX (ZERO MATRIX) is a matrix whose all elements are 0.
11. SCALAR MATRIX is a square matrix where the non – diagonal elements are 0 and all the elements on the principal diagonal are same.
12. COMPARABLE MATRICES are those matrices which have the same order. They have the same number of rows and columns. The elements may be different
13. EQUAL MATRICES are those comparable matrices having the same elements.
14. TRANSPOSE of a matrix is obtained by interchanging the rows and columns.
15. SYMMETRIC MATRIX is a matrix whose transpose is the same as original. **[A′ = A]**
16. SKEW SYMMETRIC MATRIX is that matric whose transpose is the negative of the original. **[A′ = – A]**
17. ORTHAGONAL MATRIX is a square matrix such that A A′ = A′ A = 1.
18. INVERSE of a matrix is such that the product of a matrix and its inverse is 1.
19. ADJOINT of a matrix is the transpose of the matrix of its corresponding co factors. It is written as adj. A

LAWS OF ADDITION	
A + B = B + A (Commutative)	A + (B + C) = (A + B) + C (Associative)
A + 0 = 0 + A = A	A + (– A) = (– A) + A = 0
k (A + B) = kA + kB	$(k_1 + k_2)$ A = k_1 A + k_2 A
k_1 (k_2 A) = (k_1 k_2) A	If A + B = B + C, then A = C

PROPERTIES OF MULTIPLICATION

1. 2 matrices A and B can be multiplied (AB) only if the number of columns of A is equal to the number of rows of B. If A is an (m x n) matrix and B is an (n x p) matrix than the product AB is an (m x p) matrix.
2. Matrix multiplication is not commutative in general, i.e. **AB ≠ BA**.
3. For matrices A, B, C the associative law holds true, i.e., **(A x B) x C = A x (B x C)**
4. For matrices A, B, C the distributive law over addition holds true, i.e., **A x (B + C) = (A x B) + (A x C)**
5. The product of 2 non – zero matrices can be a zero matrix.
6. If A is a given square matrix and I is an identity matrix of the same order as A then **A x I = I x A = A**
7. If A is a given square matrix and O is a null or zero matrix of the same order as A then **A x O = O x A = O**
8. **A^2 = A x A** and **A^3 = A^2 x A**

MULTIPLICATION OF 2 SQUARE MATRICES	MULTIPLICATION OF 2 RECTANGULAR MATRICES
Let A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and B = $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ Then AB = $\begin{bmatrix} (ap + br) & (aq + bs) \\ (cp + dr) & (cq + ds) \end{bmatrix}$	Let A = $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and B = $\begin{bmatrix} p & q \\ r & s \\ t & u \end{bmatrix}$ Then AB = $\begin{bmatrix} (ap + br + ct) & (aq + bs + cu) \\ (dp + er + ft) & (dq + es + fu) \end{bmatrix}$

FINDING THE INVERSE OF A MATRIX USING ELEMENTARY OPERATION

1. Write A = IA, where I is a unit matrix of the same order as A.
2. Using sequences of mathematical operations reduce LHS to I. Keep performing the same operations on RHS.
3. Obtain I = BA. Thus B is the inverse of A

FINDING THE INVERSE OF A MATRIX USING ITS ADJOINT

1. Find | A |
2. Calculate the cofactors of all the elements of A and write the matrix of these cofactors.
3. Obtain the transpose of the above matrix.
4. Find the inverse of matrix A by using the formula $A^{-1} = \frac{1}{|A|}$ x adj. A

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DETERMINANTS

1. DETERMINANT of a square matrix is a number (real or complex) that is associated to it. It is denoted by | A |. This is not read as absolute value of A.
2. If A = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then | A | = ad – bc.
3. If A = $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ then | A | = a $\begin{vmatrix} q & r \\ y & z \end{vmatrix}$ – b $\begin{vmatrix} p & r \\ x & z \end{vmatrix}$ + c $\begin{vmatrix} p & q \\ x & y \end{vmatrix}$. The value can be determined using any 1 row or column.
4. MINOR of any element T_{ij} is the determinant obtained by deleting the i th row and the j th column. It is denoted as M_{ij}.
5. COFACTOR of any element T_{ij} is $(-1)^{i+j}$ times M_{ij}. It is denoted as C_{ij}.
   [NOTE : Minors and Cofactors are defined only for square matrices.]

   PROPERTIES

6. The value of a determinant remains unchanged if its rows and columns are interchanged.
7. The value of a determinant becomes negative if any 2 rows or any columns are interchanged.
8. The value of a determinant becomes 0 if all the elements of any 1 row or column are zeroes.
9. The value of a determinant becomes 0 if any 2 rows or columns have identical elements.
10. The value of a determinant becomes k times its value when each element of any one row or column is multiplied by a scalar quantity k.
11. The value of a determinant does not change if to any row or column of the determinant a multiple of another row or column is added or subtracted.
12. If each element of a row is expressed as a sum of 2 or more terms then the determinant can also be expressed as the sum of 2 or more determinants.
13. Area of a triangle whose coordinates are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

	OPERATION	NOTATION
1	Interchanging 1st and 3rd rows	$R_1 \leftrightarrow R_3$
2	Interchanging 1st and 2nd columns	$C_1 \leftrightarrow C_2$
3	Multiplying each element of 2nd row by k	$R_2 \to kR_2$
4	Dividing each element of 3rd row by k	$R_3 \to \frac{1}{k} R_3$
5	Multiplying each element of 2nd row by k and adding it to the corresponding element of 3rd row	$R_3 \to R_3 + kR_2$
6	Multiplying each element of 2nd column by k and adding it to the corresponding element of 3rd column	$C_3 \to C_3 + kC_2$

SYSTEM OF LINEAR EQUATIONS

14. A given system of equations is said to be **CONSISTENT** if it has one or more solutions.
15. A given system of equations is said to be **INCONSISTENT** if it has no solution.
16. If $D \neq 0$, then the system is consistent and there is a unique solution and $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$.
17. If D, D_x and D_y are all equal to 0, then the system is consistent and has infinite solutions.
18. If only D = 0 and the others are non - zero, then the system is inconsistent and has no solution.

CRAMMER'S RULE

(i) Obtain D, D_x, D_y and D_z
(ii) Find the value of D.
(iii) If $D \neq 0$, then find the values of D_x, D_y and D_z. Then $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$ and $z = \frac{D_z}{D}$.
(iv) If D = 0, then find the values of D_x, D_y and D_z. If at least one of them is non – zero, then the system of linear equations is inconsistent.
(v) If $D = D_x = D_y = D_z = 0$, then shift the variable z to RHS and obtain 2 equations in x and y.
(vi) Solve the equations obtained above using Cramer's Rule and obtain the values of x and y in terms of z.
(vii) Put these values of x and y in the third equation to find the value of z.
(viii) Hence find the values of x and y.

PERMUTATIONS & COMBINATIONS

1. FUNDAMENTAL PRINCIPLE OF ADDITION states that if 2 events can be performed in m and n different ways respectively then either of the 2 events can happen in (m + n) ways.
2. FUNDAMENTAL PRINCIPLE OF MULTIPLICATION states that if an events can be performed in m different ways followed by another event that can be performed in n different ways then the 2 events can be performed in (m x n) ways.

1	$n! = n(n-1)(n-2)(n-3)\ldots \times 3 \times 2 \times 1$	2	$0! = 1$	3	$\frac{(2n)!}{n!} = 1 \times 3 \times 5 \times \ldots \times (2n-1)\ 2!$	$n! + 1$ is not divisible by $n \geq 2$

3. **PERMUTATION** is the act of arranging some or all the members of a set in a definite order by taking ***r*** things from ***n*** available things. **[{A, B} is not the same as {B, A}]**

PERMUTATION RESULTS when n and $r \in N$ such that $1 \leq r \leq n$									
1	${}^nP_r = \frac{n!}{(n-r)!}$	2	${}^nP_n = n!$	3	${}^nP_0 = 1$	4	${}^nP_1 = n$	5	${}^nP_{n-1} = n!$

4. If out of ***n*** objects ***p*** are like objects than the number of permutations of ***n*** objects taken all at a time is $\frac{n!}{p!}$
5. Number of permutations of ***n*** objects in which ***m*** objects are of one kind and the rest are of another kind is $\frac{n!}{m!(n-m)!}$
6. Number of permutations of ***n*** different objects taken ***r*** at a time when ***k*** particular objects always occur is ${}^{n-k}P_{r-k}$
7. Number of permutations of ***n*** different objects taken ***r*** at a time when ***k*** particular objects are not included is ${}^{n-k}P_r$
8. Number of circular permutations of ***n*** different objects is $(n-1)!$
9. Number of ways of arranging ***n*** different objects in a circular manner such that 2 same objects are never together is $\frac{1}{2}(n-1)!$
10. **COMBINATION** is the act of selecting some or all the members of a set in any order by taking ***r*** things from ***n*** available things. **[{A, B} is the same as {B, A}]**

COMBINATION RESULTS when n and $r \in N$ such that $1 \leq r \leq n$							
1	${}^nC_r = \frac{n!}{(n-r)!\,r!}$	3	${}^nC_r = {}^nC_{n-r}$	5	${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$	7	${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$
2	${}^nC_0 = {}^nC_n = 1$	4	${}^nC_r \times r! = {}^nP_r$	6	${}^nC_a = {}^nC_b \Rightarrow a = b$ or $a + b = n$	8	${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n - 1$

11. Number of combinations of ***n*** different objects taken ***r*** at a time when ***k*** particular objects always occur is ${}^{n-k}C_{r-k} \times r!$
12. Number of combinations of n different objects taken r at a time when k particular objects never occur is ${}^{n-k}C_r \times r!$
13. Number of ways in which ***(m + n)*** things can be divided into 2 unequal groups containing ***m*** and ***n*** things is $\frac{(m+n)!}{m!\,n!}$
14. Number of ways of distributing ***(m + n)*** things among 2 persons in unequal groups containing ***m*** and ***n*** things is $\frac{(m+n)!}{m!\,n!} \times 2!$
15. Number of ways in which **2*m*** different things can be divided into 2 equal groups each containing ***m*** things is $\frac{(2m)!}{(m!)^2\,2!}$
16. Number of ways in which **3*m*** different things can be divided into 3 equal groups each containing ***m*** things is $\frac{(3m)!}{(m!)^3\,3!}$
17. Number of ways in which **2*m*** different things can be distributed among 2 persons in equal groups each containing ***m*** things is $\frac{(2m)!}{(m!)^2}$
18. Number of ways in which 3*m* different things can be distributed among 3 persons in equal groups each containing ***m*** things is $\frac{(3m)!}{(m!)^3}$
19. Number of ways of distributing ***n*** identical objects to ***r*** persons each one of whom gets 0, 1, 2 or more objects is ${}^{n+r-1}C_{r-1}$
20. Number of ways of distributing ***n*** identical objects to ***r*** persons each one of whom gets at least one objects is ${}^{n-1}C_{r-1}$
21. Number of straight lines that can be formed joining ***n*** points if ***p*** points are collinear is ${}^nC_2 - {}^pC_2 + 1$.
22. Number of triangles that can be formed joining ***n*** points if ***p*** points are collinear is ${}^nC_3 - {}^pC_3$.

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TRIGONOMETRY

1. TRIGONOMETRY is that branch of mathematics which deals with the measurements of the sides and the angles of a triangle.
2. TRIGONOMETRIC RATIOS are the values of all the trigonometric functions based on the value of the ratio of the sides in a right angled triangle.

TRIGONOMETRIC RATIOS					
i	$\sin\theta = \frac{\text{Perpendicular (Opposite)}}{\text{Hypotenuse}} = \frac{1}{\text{cosec}\,\theta}$	ii	$\cos\theta = \frac{\text{Base (Adjacent)}}{\text{Hypotenuse}} = \frac{1}{\sec\theta}$	iii	$\tan\theta = \frac{\text{Perpendicular (Opposite)}}{\text{Base (Aadjacent)}} = \frac{1}{\cot\theta}$
iv	$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular (Opposite)}} = \frac{1}{\sin\theta}$	v	$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base (Adjaceent)}} = \frac{1}{\cos\theta}$	vi	$\cot\theta = \frac{\text{Base (Adjacent)}}{\text{Perpendicular (Opposite)}} = \frac{1}{\tan\theta}$
REMEMBER	***S**ome **P**eople **H**ave / **C**urly **B**lack **H**air / **T**urning **P**robably **B**rown* or OSHACHOTA or SOH CAH TOA				

SINE RULE is $\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$	**COSINE RULE** is $a^2 = b^2 + c^2 - 2bc\cos A$	*a, b, c are sides of a triangle and A, B, C are angles*

TRIGONOMETRIC IDENTITIES					
1	$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$	2	$\text{Sec}^2\theta - \text{Tan}^2\theta = 1$	3	$\text{Cosec}^2\theta - \text{Cot}^2\theta = 1$
	$\text{Sin}^2\theta = 1 - \text{Cos}^2\theta$		$\text{Sec}^2\theta = 1 + \text{Tan}^2\theta$		$\text{Cosec}^2\theta = 1 + \text{Cot}^2\theta$
	$\text{Cos}^2\theta = 1 - \text{Sin}^2\theta$		$\text{Tan}^2\theta = \text{Sec}^2\theta - 1$		$\text{Cot}^2\theta = \text{Cosec}^2\theta - 1$

TRIGONOMETRIC RATIOS OF PARTICULAR ANGLES

	0°	15°	30°	45°	60°	75°	90°
Sin θ	0	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	1
Cos θ	1	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	0
Tan θ	0	$2-\sqrt{3}$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$2+\sqrt{3}$	∞
Cosec θ	∞	$\frac{2\sqrt{2}}{\sqrt{3}-1}$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	$\frac{2\sqrt{2}}{\sqrt{3}+1}$	1
Sec θ	1	$\frac{2\sqrt{2}}{\sqrt{3}+1}$	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\frac{2\sqrt{2}}{\sqrt{3}-1}$	∞
Cot θ	∞	$2+\sqrt{3}$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	$2-\sqrt{3}$	0

TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES		
sin (90° – θ) = cos θ	**tan (90° – θ) = cot θ**	**sec (90° – θ) = cosec θ**
cos (90° – θ) = sin θ	**cot (90° – θ) = tan θ**	**cosec (90° – θ) = sin θ**

180° = π radians	1. TO CONVERT DEGREES INTO RADIANS MULTIPLY BY $\frac{\pi}{180}$, KEEPING π AS IT IS 2. TO CONVERT RADIANS INTO DEGREES MULTIPLY BY $\frac{180}{\pi}$, CONVERT π TO $\frac{22}{7}$

ALLIED ANGLES

Function	θ = 90 – α	θ = 90 + α	θ = 180 – α	θ = 180 + α	θ = 270 – α	θ = 270 + α	θ = 360 – α	θ = 360 + α
Sin θ	Cos α	Cos α	Sin α	– Sin α	– Cos α	– Cos α	– Sin α	Sin α
Cos θ	Sin α	– Sin α	– Cos α	– Cos α	– Sin α	Sin α	Cos α	Cos α
Tan θ	Cot α	– Cot α	– Tan α	Tan α	Cot α	– Cot α	– Tan α	Tan α
Cosec θ	Sec α	Sec α	Cosec α	– Cosec α	– Sec α	– Sec α	– Tan α	Tan α
Sec θ	Cosec α	– Cosec α	– Sec α	– Sec α	– Cosec α	Cosec α	Sec α	Sec α
Cot θ	Tan α	– Tan α	– Cot α	Cot α	Tan α	– Tan α	– Cot α	Cot α
	Q I	**Q II**		**Q III**		**Q IV**		**Q I**

VIJAYGIRI MATH CLASS

FORMULAE SHEET

COMPOUND ANGLES		
$\sin(A+B) = \sin A \cos B + \cos A \sin B$	$\cos(A+B) = \cos A \cos B - \sin A \sin B$	$\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$
$\sin(A-B) = \sin A \cos B - \cos A \sin B$	$\cos(A-B) = \cos A \cos B + \sin A \sin B$	$\cos^2 B - \cos^2 A = \sin(A+B)\sin(A-B)$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	$\sin(A+B)\sin(A-B) = (\sin A - \sin B)(\sin A + \sin B)$
$2\cos A \sin B = \sin(A+B) - \sin(A-B)$	$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	$\cos(A+B)\cos(A-B) = (\cos A + \sin B)(\cos A - \sin B)$
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$	$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$	$\tan(A+B)\tan(A-B) = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$
$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$	$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$	$\tan A + \tan B = \tan(A+B)(1 - \tan A \tan B)$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$	$\tan A - \tan B = \tan(A-B)(1 + \tan A \tan B)$
$\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}$	$\tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B}$	$\cos^2 A - \cos^2 B = (\cos A + \cos B)(\cos A - \cos B)$

DOUBLE ANGLE	TRIPLE ANGLE	HALF ANGLE
$\sin 2A = 2\sin A \cos A = \frac{2\tan A}{1+\tan^2 A}$	$\sin 3A = 3\sin A - 4\sin^3 A$	$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
$\cos 2A = \cos^2 A - \sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$	$\cos 3A = 4\cos^3 A - 3\cos A$	$\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} = \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
$\cos 2A = 1 - 2\sin^2 A = 2\cos^2 A - 1$		$\cos A = 1 - 2\sin^2\frac{A}{2} = 2\cos^2\frac{A}{2} - 1$
$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$	$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$	$\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$
$1 + \sin 2A = (\cos A + \sin A)^2$	$1 + \cos 2A = 2\cos^2 A$	$\frac{1+\sin 2A}{\cos 2A} = \frac{\cos A + \sin A}{\cos A - \sin A}$
$1 - \sin 2A = (\cos A - \sin A)^2$	$1 - \cos 2A = 2\sin^2 A$	$\frac{1-\sin 2A}{\cos 2A} = \frac{\cos A - \sin A}{\cos A + \sin A}$
$1 + \sin A = (\cos\frac{A}{2} + \sin\frac{A}{2})^2$	$1 + \cos A = 2\cos^2\frac{A}{2}$	$\frac{1+\cos 2A}{\sin 2A} = \cot A$
$1 - \sin A = (\cos\frac{A}{2} - \sin\frac{A}{2})^2$	$1 - \cos A = 2\sin^2\frac{A}{2}$	$\frac{1-\cos 2A}{\sin 2A} = \tan A$
$\frac{1+\sin 2A}{1-\sin 2A} = \tan^2(\frac{\pi}{4} + A)$	$\frac{1+\cos 2A}{1-\cos 2A} = \cot^2 A$	$\frac{1+\tan A}{1-\tan A} = \tan(\frac{\pi}{4} + A)$
$\frac{1-\sin 2A}{1+\sin 2A} = \tan^2(\frac{\pi}{4} - A)$	$\frac{1-\cos 2A}{1+\cos 2A} = \tan^2 A$	$\frac{1-\tan A}{1+\tan A} = \tan(\frac{\pi}{4} - A)$

INVERSE TRIGONOMETRY			
$\sin^{-1}(\sin x) = x$	$\cos^{-1}(\cos x) = x$	$\tan^{-1}(\tan x) = x$	$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
$\sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1} x$	$\cos^{-1}\frac{1}{x} = \sec^{-1} x$	$\tan^{-1}\frac{1}{x} = \cot^{-1} x$	$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
$\sin^{-1}(-x) = -\sin^{-1} x$	$\cos^{-1}(-x) = \pi - \cos^{-1} x$	$\tan^{-1}(-x) = \tan^{-1} x$	$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$
$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$	$\sec^{-1}(\sec x) = x$	$\cot^{-1}(\cot x) = x$	$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}$
$\operatorname{cosec}^{-1}\frac{1}{x} = \sin^{-1} x$	$\sec^{-1}\frac{1}{x} = \cos^{-1} x$	$\cot^{-1}\frac{1}{x} = \tan^{-1} x$	$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}$
$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$	$\sec^{-1}(-x) = \pi - \sec^{-1} x$	$\cot^{-1}(-x) = \pi - \cot^{-1} x$	

$\sin^{-1} x + \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$	$\sin^{-1} x - \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$
$\cos^{-1} x + \cos^{-1} y = \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}$	$\cos^{-1} x - \cos^{-1} y = \cos^{-1}(xy + \sqrt{1-x^2}\sqrt{1-y^2}$
$2\sin^{-1} x = \sin^{-1}[2x\sqrt{1-x^2})$ $\quad \frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$	$2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$ $\quad \frac{1}{\sqrt{2}} \le x \le 1$
$3\sin^{-1} x = \sin^{-1}[3x - 4x^3]$ $\quad x \in [\frac{-1}{2}, \frac{1}{2}]$	$3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$ $\quad x \in [\frac{1}{2}, 1]$

$$2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}, |x| < 1 \quad = \sin^{-1}\frac{2x}{1+x^2}, |x| < 1 \quad = \cos^{-1}\frac{1-x^2}{1+x^2}, |x| \ge 0$$

NOTE : $\sin^{-1} x \ne (\sin x)^{-1}$; $(\sin^{-1} x)^2 \ne \sin^{-2} x$, etc. for ALL TRIGONOMETRIC FUNCTIONS

NOTE

*When there is a linear **cosine** and a linear **sine** function in the numerator and the denominator such that the cosine function is positive, their coefficients are one and the angle is the same then divide the numerator and denominator by **cos (angle)***

EXAMPLE : $\frac{\cos A + \sin A}{\cos A - \sin A} = \frac{\frac{\cos A + \sin A}{\cos A}}{\frac{\cos A - \sin A}{\cos A}} = \frac{1 + \tan A}{1 - \tan A} = \tan(\frac{\pi}{4} - A)$

HEIGHTS & DISTANCES

1. ANGLE OF ELEVATION is the angle formed between the horizontal line and the oblique line of sight for an object situated above.
2. ANGLE OF DEPRESSION is the angle formed between the horizontal line and the oblique line of sight for an object situated below.
3. LINE OF SIGHT is the oblique line drawn from the observer's eye to the point being viewed on the object.
4. HORIZONTAL LINE is an imaginary line assumed to be at the horizontal eye level of the observer.
5. If A and B are two points on the **same side** of a tower or building of height h such that AB = d and the angles of elevation of the top of the tower from A and B are α and β, then : (i) $\mathbf{h = \frac{d \tan\alpha \tan\beta}{\tan\alpha - \tan\beta}}$ (ii) $\mathbf{d = h (\cot\beta - \cot\alpha)}$ given that $\alpha > \beta$.
6. If A and B are two points on **opposite sides** of a tower or building of height h such that AB = d and the angles of elevation of the top of the tower from A and B are α and β, then : (i) $\mathbf{h = \frac{d \tan\alpha \tan\beta}{\tan\alpha + \tan\beta}}$ (ii) $\mathbf{d = h (\cot\beta + \cot\alpha)}$ given that $\alpha > \beta$.
7. HEIGHT OF A VERTICAL TOWER surmounted by a flag staff or a tank of height h such that the angles of elevation of the top and bottom of the flag staff or tank are α and β respectively is $\frac{h \tan\beta}{\tan\alpha - \tan\beta}$.
8. HEIGHT OF A FLAG STAFF OR A TANK atop a vertical tower of height h such that the angles of elevation of the top and bottom of the flag staff or tank are α and β respectively is $\mathbf{h (\tan\alpha \cot\beta - 1)}$.
9. HEIGHT OF A VERTICAL TOWER at a distance a and b units from two points such that the angles of elevation of the top of the tower from these two points are complementary is $\sqrt{ab}$ units.
10. HEIGHT OF A CLOUD when it is given that its angle of elevation from a point h units above a lake is α and the angle of depression of its reflection in the lake is β is $\frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$.

GEOMETRY

1. POINT is just a location marker showing the exact position of an object and is represented by a dot. It has no length, width or thickness.
2. LINE is a set of points extending infinitely on both sides. It has no width and is absolutely straight. It is denoted as a straight line with arrows at both ends.
3. LINE SEGMENT is a portion of the straight line between 2 fixed points. It is denoted as a straight line with no arrows.
4. COLLINEAR POINTS are 3 or more than 3 points lying in a straight line.
5. RAY is a line with one end being fixed.
6. PLANE is a flat smooth surface that has length and width but no height. It extends infinitely in all directions.
7. PARALLEL LINES are 2 or more lines lying in the same plane which do not intersect each other at any point.
8. CONCURRENT LINES are 3 or more straight lines passing through the same point.
9. ANGLE is formed by 2 rays with the same initial point. The different angles are :

1	Acute angle which is more than 0° but less than 90°	2	Right angle which is exactly 90°
3	Obtuse angle which is more than 90° but less than 180°	4	Straight angle which is exactly 180°
5	Reflex angle which is more than 180° but less than 270°	6	Complete angle which is exactly 360°

10. COMPLEMENTARY ANGLES are those 2 angles whose sum is 90°
11. SUPPLEMENTARY ANGLES are those 2 angles whose sum is 180°
12. ADJACENT ANGLES are those 2 angles which have a common arm and the other arms are on the opposite sides of the common arm.
13. ALTERNATE ANGLES are those two non – adjacent angles which are formed by the parallel lines and the transversal and which lie on opposite sides of the transversal either in the exterior or in the interior of the parallel lines. They are of equal measures.
14. CORRESPONDING ANGLES are those two non – adjacent angles which are formed by the parallel lines and the transversal and lie on the same side of the transversal with one lying in the exterior and the other in the interior of the parallel lines. They are of equal measures.

15. CO INTERIOR ANGLES are those two non – adjacent angles which are formed by the parallel lines and the transversal and lie on the same side of the transversal and are interior. Their sum is 180°.
16. VERTICALLY OPPOSITE ANGLES are a pair of angles that are opposite each other at a vertex when two straight lines intersect. They are of equal measures.
17. LINEAR PAIR is a pair of adjacent angles which add up to 180° and form a straight line.
18. TRIANGLE is formed when 3 straight lines meet each other at 3 vertices. A triangle will consist of 3 sides, 3 vertices and 3 angles. The sum of all the 3 angles is exactly 180° and the sum of any 2 sides of the triangle is always greater than the third side. The different types of triangles are :

ACCORDING TO SIDES		**ACCORDING TO ANGLES**	
Scalene	All 3 sides are of different measure	**Acute angled**	Measure of each angle is less than 90°.
Isosceles	Any 2 sides have the same measure	**Right angled**	Only 1 angle is 90° and the other 2 are acute
Equilateral	All 3 sides have the same measure.	**Obtuse angled**	Only 1 angle is obtuse and other 2 are acute

PROPERTIES OF A TRIANGLE	
1	The sum of all the three interior angles of any triangle is always 180°
2	The sum of all the three exterior angles of any triangle is always 360°
3	The sum of an interior angle and its corresponding exterior is always 180°
4	The exterior angle of any triangle is equal to the sum of its interior opposite angles.
5	The sum of any two sides of a triangle is always greater than its third side
6	The difference between any two sides of a triangle is less than the third side.
7	The side opposite to the largest angle is always the largest side and vice versa.
8	In a right angled triangle the side opposite to the right angle is called the hypotenuse.
9	If the bisector of the vertex angle of a triangle bisects the base than the triangle is isosceles.
10	Medians in an equilateral triangle are always perpendicular to their base.
11	The perimeter of a triangle is greater than the sum of the three altitudes or the medians.
12	The sum of any 2 sides of a triangle is greater than twice the median drawn on the third side.

19. MEDIAN of a triangle is a line segment that connects the triangle's vertex to the midpoint of the opposite side.
20. CEVIAN is a line segment that joins the vertex of a triangle to a point on the opposite side.
21. CENTROID of a triangle is the point of intersection of the three medians. It divides the median in the ratio 2 : 1.
22. INCENTRE of a triangle is the point of intersection of the three angle bisectors. It is equidistant from all sides.
23. CIRCUMCENTRE of a triangle is the point of intersection of the perpendicular bisectors of the sides. It is equidistant from all the 3 vertices.
24. ORTHOCENTRE of a triangle is the point of intersection of the altitudes drawn from the vertices of the triangle to the opposite sides.
    a) For an acute angled triangle it will be inside the triangle
    b) For an obtuse angled triangle it will be outside the triangle.
    c) For a right angled triangle it will lie on the vertex of the triangle.
25. **CONGRUENT TRIANGLES** are those 2 triangles which have the same shape and size. Their corresponding sides and their corresponding angles are equal in measure. The different congruency conditions are :

SSS	If 3 sides of one triangle are equal to the 3 sides of another triangle then the two triangles are congruent.
SAS	If 2 sides and the included angle of one triangle are equal in measure to the corresponding 2 sides and the included angle of the other, then the two triangles are congruent.
ASA	If 2 angles and the included side of one triangle are equal in measure to the corresponding 2 angles and the included side of the other, then the two triangles are congruent.
AAS	If 2 angles and the non included side of one triangle are equal in measure to the corresponding 2 angles and the non included side of the other, then the two triangles are congruent.
RHS	2 right angles triangles are congruent if the hypotenuse and one side of one triangle are equal to the hypotenuse and the corresponding side of the other triangle.

26. **SIMILAR TRIANGLES** are those 2 triangles whose corresponding angles are equal and corresponding sides are proportional.
27. **MIDPOINT THEOREM** states that the line joining the midpoints of any 2 sides of a triangle is parallel to the third side and half of it.
28. **BASIC PROPORTIONALITY THEOREM (THALES THEOREM)** states that if a line is drawn parallel to a given side of a triangle than it divides the other two sides in the same ratio.
29. **TRANSVERSAL THEOREM** states that if 3 or more parallel lines are intersected by 2 transversals than the intercepts made by them on the transversals are proportional.

30. **ANGLE BISECTOR THEOREM** states that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angles.
31. **PYTHAGORAS THEOREM** states the square of the hypotenuse is equal to the sum of the squares of the other two sides.
32. **APPLLONIUS THEOREM** states that the sum of the squares of any 2 sides of a triangle is equal to twice the square of half the third side plus twice the square of the median that bisects the third side.

THEOREMS ON SIMILAR TRIANGLES		
1	If two triangles are similar then the ratio of their corresponding sides are equal	$\Delta ABC \sim \Delta PQR \Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$
2	If two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides	$\Delta ABC \sim \Delta PQR \Rightarrow \frac{\text{Area of ABC}}{\text{Area of PQR}} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$
3	If two triangles are similar then the ratio of their medians is equal to the ratio of their corresponding sides	$\Delta ABC \sim \Delta PQR \Rightarrow \frac{AD}{PT} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$
4	If two triangles are similar then the ratio of their area is equal to the ratio of the squares of their corresponding sides	$\Delta ABC \sim \Delta PQR \Rightarrow \frac{\text{Area of ABC}}{\text{Area of PQR}} = \frac{(AD)^2}{(PT)^2}$
5	If two triangles are similar then the ratio of their perimeters is equal to the ratio of their corresponding sides	$\Delta ABC \sim \Delta PQR \Rightarrow \frac{\text{Perimeter of ABC}}{\text{Perimeter of PQR}} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

AREA OF A TRIANGLE

1. Using **AREA** formula : **Area** = $\frac{1}{2}$ **base x height**
2. Using **HERON'S** formula : **Area** = $\sqrt{s\,(s-a)(s-b)(s-c)}$, where s is the semi perimeter of the triangle whose sides are *a*, *b* and *c*. $s = \frac{\text{Perimeter}}{2} = \frac{a+b+c}{2}$
3. Using **TRIGONOMETRY** : **Area** = $\frac{1}{2}ab\sin C$ where *a*, *b* are the sides of a triangle and C is the included angle.

33. POLYGON is a 2 dimensional closed figure made up of a minimum of 3 line segments. They are classified on the number sides, angles and whether they are regular or irregular.

	POLYGON	No. of sides	Sum of angles		POLYGON	No. of sides	Sum of angles
1	Triangle	3	180°	6	Octagon	8	1080°
2	Quadrilateral	4	360°	7	Nonagon	9	1260°
3	Pentagon	5	540°	8	Decagon	10	1440°
4	Hexagon	6	720°	9	Hendecagon	11	1620°
5	Heptagon	7	900°	10	Dodecagon	12	1800°

34. REGULAR POLYGON is that in which all sides and angles are equal.
35. IRREGULAR POLYGON is that in which all sides and angles are not equal.
36. The sum of all the exterior angles of any polygon is always 360°
37. Formula to calculate the interior angle of a regular polygon is $\frac{(n-2) \times 180}{n}$
38. Formula to calculate the exterior angle of a regular polygon is $\frac{360}{n}$
39. QUADRILATERAL is a closed two dimensional shape and a type of polygon that has 4 sides, 4 vertices and 4 angles. The sum of all its interior angles is always 360°

TYPES OF QUADRILATERALS			
NAME	**PROPERTIES**	**NAME**	**PROPERTIES**
Parallelogram	1. Opposite sides are equal and parallel 2. Opposite angles are equal. 3. Adjacent angles are supplementary 4. Diagonals are not equal but they bisect each other.	Rhombus	1. All sides are equal. 2. Opposite sides are parallel and angles are equal. 3. Adjacent angles are supplementary. 4. Diagonals are not equal but they bisect each other at right angles.
Rectangle	1. Opposite sides are equal and parallel. 2. Each angle is 90° 3. Diagonals are equal and bisect each other.	Trapezium	1. One pair of opposite sides are parallel. 2. Adjacent angles are supplementary. 3. Diagonals intersect but do not bisect each other.
Square	1. Opposite sides are parallel. 2. All sides are equal. 3. Each angle is 90° 4. Diagonals are equal and bisect each other at 90°	Cyclic Quadrilateral	1. All the 4 vertices lie on the circumference of a circle. 2. Opposite angles are supplementary. 3. The exterior angle is equal to the interior opposite angle. 4. Perpendicular bisectors of the sides meet at the center of the circle.
Kite	1. 2 pairs of adjacent sides are equal. 2. The diagonals are not equal but they intersect each other at 90°	Arrow	1. 2 pairs of adjacent sides are equal. 2. Diagonals bisect externally and are perpendicular to each other. 3. One interior angle is reflex.

40. CIRCLE is defined the locus of a point which moves in a plane such that its distance from a fixed point is constant. The various parts of a circle are : • Centre • Radius • Diameter • Circumference • Chord • Tangent • Secant • Arc • Segment • Sector

NOTE	1. Diameter is twice the radius and the longest chord of the circle 2. Tangent touches the circle at only one point 3. Secant crosses the circle at two points.

41. CONCENTRIC CIRCLES are those circles which have the same centre but different radii.
42. CONCYLIC POINTS are those three or more points which lie on the same circle.
43. STANDARD EQUATION OF A CIRCLE is $\mathbf{(x - h)^2 + (y - k)^2 = a^2}$, where the centre of the circle is (h, k) and the radius is a.
44. EQUATION OF A CIRCLE WHEN THE CENTRE IS THE ORIGIN is $\mathbf{x^2 + y^2 = a^2}$.
45. EQUATION OF A CIRCLE PASSING THROUGH THE ORIGIN is $\mathbf{x^2 + y^2 - 2hx - 2ky = 0}$ where (h, k) is the centre of the circle.
46. EQUATION OF A CIRCLE TOUCHING THE X – AXIS is $\mathbf{x^2 + y^2 - 2hx - 2ay + h^2 = 0}$, where (h, k) is the centre of the circle.
47. EQUATION OF A CIRCLE TOUCHING THE Y – AXIS is $\mathbf{x^2 + y^2 - 2ax - 2ky + k^2 = 0}$, where (h, k) is the centre of the circle.
48. EQUATION OF A CIRCLE TOUCHING BOTH AXIS is $\mathbf{x^2 + y^2 - 2ax - 2ay + a^2 = 0}$, where (h, k) is the centre of the circle.
49. EQUATION OF A CIRCLE PASSING THROUGH THE ORIGIN AND CENTRE LYING ON THE X – AXIS is $\mathbf{x^2 + y^2 - 2ax = 0}$, where (h, 0) is the center of the circle.
50. EQUATION OF A CIRCLE PASSING THROUGH THE ORIGIN AND CENTRE LYING ON THE Y – AXIS is $\mathbf{x^2 + y^2 - 2ay = 0}$, where (0, k) is the center of the circle.
51. EQUATION OF A CIRCLE DRAWN ON A DIAMETER WHOSE END POINTS ARE (x_1, y_1) AND (x_2, y_2) is $\mathbf{(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0}$
52. GENERAL EQUATION OF A CIRCLE is $\mathbf{x^2 + y^2 + 2gx + 2fy + c = 0}$, where the centre of the circle is (– g, – f) and the radius is $\sqrt{\mathbf{g^2 + f^2 - c}}$

NOTE	1. If $g^2 + f^2 - c > 0$, then the radius of the circle is real and the circle is also real. 2. If $g^2 + f^2 - c = 0$, then the radius of the circle is 0 and the circle is a point circle. 3. If $g^2 + f^2 - c < 0$, then the radius of the circle is imaginary but the circle is real.

CIRCLE THEOREMS	
Theorem 1	Equal chords of a circle subtend equal angles at the centre.
Theorem 2	Corresponding chords of congruent arcs are equal
Theorem 3	Perpendicular from the center of a circle bisects the chord.
Theorem 4	Equal chords of a circle are equidistant from the centre.
Theorem 5	If 2 arcs of congruent circles are congruent, then the corresponding chords are equal.
Theorem 6	Equal chords of congruent circles are equidistant from their corresponding centres.
Theorem 7	Equal chords of congruent circles su btend equal angles at the centre.
Theorem 8	Angle subtended by an arc at the centre is twice the angle subtended by it on the circumference.
Theorem 9	Angles on the same base and in the same segment are equal
Theorem 10	Angle in a semi – circle is always a right angle.
Theorem 11	Opposite angles of a cyclic quadrilateral are supplementary.
Theorem 12	The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.
Theorem 13	If two chords of a circle, AB and CD intersect either internally or externally at P, then **PA x PB = PC x PD**.
Theorem 14	The tangent at any point of a circle is perpendicular to the radius at the point of contact.
Theorem 15	Tangents to a circle from the same point are equal in length.
Theorem 16	The angle between the tangents is supplementary to the angle subtended by them at the centre.
Theorem 17	If a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent. **[PA x PB = PT²]**
Theorem 18	The angle formed by a chord and a tangent of the same circle is equal to the angle in the alternate segment.

53. LOCUS is the path taken by a moving point satisfying given geometrical conditions.

LOCI THEOREMS	
Theorem 1	Locus of a point equidistant from a fixed point is a circle with the fixed point as centre.
Theorem 2	Locus of a point equidistant from 2 fixed points lies on the perpendicular bisector of the line segment joining the 2 fixed points.
Theorem 3	Locus of a point from the arms of an angle lies on the angle bisector.
Theorem 4	Locus of a point equidistant from a given straight line lies on a line parallel to the given straight line.
Theorem 5	Locus of a Parabola is the path taken by a point such that it is equidistant from the Focus and the Directrix
Theorem 6	Locus of an Ellipse is the path taken by a point such that the sum of the distances of 2 focal points is fixed.
Theorem 7	Locus of a Hyperbola is the path taken by a point such that the absolute value of the difference between the distances of two given foci is always constant.

CONIC SECTION

1. CONIC SECTION is the locus of a point P which moves in such a way that its distance from a fixed point S always bears a constant ratio to its distance from a fixed line, all being in the same plane.
2. FOCUS is the fixed point of a conic section.
3. DIRECTRIX is the fixed straight line of a conic section.

NOTE	1. Every conic section has 4 foci, 2 real and 2 imaginary. 2. Every conic section has 2 directrices corresponding to the 2 real foci.

4. ECCENTRICITY is the constant ratio of a conic section and is denoted by e.

NOTE		
	1. For $e < 1$, the conic obtained is an ellipse.	2. For $e = 1$, the conic obtained is a parabola.
	3. For $e > 1$, the conic obtained is a hyperbola.	4. For $e = 0$, the conic obtained is a circle.

5. AXIS of a conic section is the straight line passing through the focus and perpendicular to the directrix.
6. VERTEX of a conic section is the point of intersection of the conic section and the axis.
7. CENTRE of a conic section is a point which bisects every chord of the conic section passing through it.
8. LATUS RECTUM of a conic section is the chord passing through the focus and perpendicular to the axis.
9. PARABOLA is the locus of a point which moves in a plane such that its distance from a fixed point in the plane is always equal to its distance from a fixed straight line in the same plane.
10. EQUATION OF A PARABOLA is $\mathbf{y^2 = 4ax}$. It can also be $\mathbf{y^2 = -4ax}$ or $\mathbf{x^2 = 4ay}$ or $\mathbf{x^2 = -4ay}$

STANDARD FORMS OF PARABOLA				
	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Coordinates of the Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Coordinates of Focus	(a, 0)	(– a, 0)	(0, a)	(0, – a)
Equation of the Directrix	x = – a	x = a	y = – a	y = a
Equation of the Axis	y = 0	y = 0	x = 0	x = 0
Length of the Latus Resctum	4a	4a	4a	4a
Focal distance of point P (x, y)	a + x	a – x	a + y	a – y

11. ELLIPSE is the locus of a point in a plane which moves in such a way that the ratio of its distance from a fixed point (focus) to its distance from a fixed straight line (directrix) is always constant and less than unity.
12. MAJOR AXIS is the longest diameter of the ellipse going through the centre from one end to the other.
13. MINOR AXIS is the shortest diameter of the ellipse crossing through the centre at the narrowest part.
14. EQUATION OF AN ELLIPSE is $\mathbf{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$ where a > b or $\mathbf{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$ where a < b

STANDARD FORMS OF AN ELLIPSE					
	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$		$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
Coordinates of the centre	(0, 0)	(0, 0)	Coordinates of the vertices	(a, 0) & (– a, 0)	(0, b) & (0, – b)
Coordinates of the foci	(ae, 0) & (- ae, 0)	(0, be) & (0, – be)	Focal distance of a point	$a \pm ex$	$b \pm ey$
Length of Major Axis	2a	2b	Length of Minor Axis	2b	2a
Equation of Major Axis	y = 0	x = 0	Equation of Minor Axis	x = 0	y = 0
Eccentricity	$\sqrt{1 - \frac{b^2}{a^2}}$	$\sqrt{1 - \frac{a^2}{b^2}}$	Equation of the directrices	$x = \frac{a}{e}$ or $\frac{-a}{e}$	$y = \frac{b}{e}$ or $\frac{-b}{e}$
Length of Latus Rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$			

15. HYPERBOLA is the locus of a point in a plane which moves in such a way that the ratio of its distance from a fixed point (focus) to its distance from a fixed straight line (directrix) is always constant and greater than unity.
16. EQUATION OF A HYPERBOLA is $\mathbf{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$ or $\mathbf{\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1}$, where $b^2 = a^2(e^2 - 1)$
17. TRANSVERSE AXIS is the line passing through the 2 foci and the centre of the hyperbola.
18. CONJUGATE AXIS is the line passing through the centre of the hyperbola and perpendicular to the transverse axis.

STANDARD FORMS OF A HYPERBOLA					
	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$		$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
Coordinates of the centre	(0, 0)	(0, 0)	Coordinates of the vertices	(a, 0) & (– a, 0)	(0, b) & (0, – b)
Coordinates of the foci	($\pm ae$, 0)	(0, $\pm be$)	Focal distance of a point	$ex \pm a$	$ey \pm b$
Length of Transverse Axis	2a	2b	Length of Conjugate Axis	2b	2a
Equation of Major Axis	y = 0	x = 0	Equation of Minor Axis	x = 0	y = 0
Eccentricity	$e = \frac{\sqrt{a^2 + b^2}}{b}$	$e = \frac{\sqrt{a^2 + b^2}}{a}$	Equation of the directrices	$x = \pm\frac{a}{e}$	$y = \pm\frac{b}{e}$
	$b^2 = a^2(e^2 - 1)$	$a^2 = b^2(e^2 - 1)$			
Length of Latus Rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$			

FORMULAE SHEET

COORDINATE GEOMETRY

1. Coordinates of a point lying on the x – axis are $(k, 0)$.
2. Coordinates of a point lying on the y – axis are $(0, k)$.
3. SECTION FORMULA : If a line segment whose end points are A (x_1, y_1) and B (x_2, y_2) is divided internally at point P (x, y) in the ratio m : n then $\mathbf{x} = \frac{mx_2 + nx_1}{m + n}$ and $\mathbf{y} = \frac{my_2 + ny_1}{m + n}$
4. SECTION FORMULA : If a line segment whose end points are A (x_1, y_1) and B (x_2, y_2) is divided externally at point P (x, y) in the ratio m : n then $\mathbf{x} = \frac{mx_2 - nx_1}{m - n}$ and $\mathbf{y} = \frac{my_2 - ny_1}{m - n}$
5. MIDPOINT FORMULA : If P (x, y) is the midpoint of the line segment joining the points A (x_1, y_1) and B (x_2, y_2), then $\mathbf{x} = \frac{x_2 + x_1}{2}$ and $\mathbf{y} = \frac{y_2 + y_1}{2}$
6. DISTANCE FORMULA : The distance between 2 points (x_1, y_1) and (x_2, y_2) is $\mathbf{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
7. AREA OF TRIANGLE is $\frac{1}{2}\,[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$
8. CENTROID OF A TRIANGLE whose coordinates are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are $\mathbf{x} = \frac{x_1 + x_2 + x_3}{3}$ and $\mathbf{y} = \frac{y_1 + y_2 + y_3}{3}$
9. INCENTRE OF A TRIANGLE whose coordinates are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are $\mathbf{x} = \frac{ax_1 + bx_2 + cx_3}{a + b + c}$ and $\mathbf{y} = \frac{ay_1 + by_2 + cy_3}{a + b + c}$, where a = BC, b = AC and c = AB
10. GENERAL EQUATION OF A STRAIGHT LINE is $\mathbf{ax + by + c = 0}$
11. SLOPE INTERCEPT EQUATION of a straight line is $\mathbf{y = mx + c}$, where m is the slope of the line and c is the y intercept.
12. POINT SLOPE EQUATION of a straight line is $\mathbf{y - y_1 = m (x - x_1)}$ where m is the slope and the point is (x_1, y_1).
13. POINT – POINT EQUATION of a straight line is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$, where the given points are A (x_1, y_1) and B (x_2, y_2).
14. INTERCEPT EQUATION of a line is $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are the intercepts made by the line on the coordinate axis.
15. SLOPE (m) OF A STRAIGHT LINE having points A (x_1, y_1) and B (x_2, y_2) on it is $\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1}$
16. SLOPES OF PARALLEL LINES are equal. Thus $\mathbf{m_1 = m_2}$
17. SLOPES OF PERPENDICULAR LINES are negative reciprocals of each other. Thus $\mathbf{m_1 m_2 = -1}$
18. SLOPE OF A HORIZONTAL LINE is 0
19. SLOPE OF A VERTICAL LINE is undefined.
20. EQUATION OF A LINE PARALLEL TO THE X – AXIS is $\mathbf{y = \pm k}$
21. EQUATION OF A LINE PARALLEL TO THE Y – AXIS is $\mathbf{x = \pm k}$
22. ANGLE (θ) BETWEEN 2 LINES is $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$
23. PERPENDICULAR FORM OF A LINE is $\mathbf{x\cos\alpha + y\sin\alpha = p}$, where p is the length of the perpendicular and α is the angle made by the perpendicular with the x – axis.
24. DISTANCE FORM OF A LINE is $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = \mathbf{r}$, where the line is passing through (x_1, y_1), making an angle θ with the x – axis and r is the distance of the point from (x_1, y_1).
25. DISTANCE OF A POINT FROM A GIVEN LINE is $\mathbf{d} = \left|\frac{a x_1 + b y_1 + c}{\sqrt{a^2 + b^2}}\right|$, the point is (x_1, y_1) and the given straight line is ax + by + c = 0
26. DISTANCE BETWEEN 2 PARALLEL LINES is $\mathbf{d} = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.
27. EQUATION OF A LINE passing through (x_1, y_1) and making an angle θ with the line y = mx + c is $\mathbf{y - y_1} = \frac{m \pm \tan\theta}{1 \pm m\tan\theta}\,\mathbf{(x - x_1)}$

FORMULAE SHEET

STATISTICS

1. STATISTICS is that branch of mathematics that deals with the collection, presentation, analysis and interpretation of numerical data.
2. PRIMARY DATA is the data collected by the investigator personally. It is highly reliable and relevant.
3. SECONDARY DATA is the data collected by someone other than the investigator.
4. RAW (UNGROUPED) DATA is the data obtained in original form.
5. VARIATE is any character capable of taking several different values.
6. RANGE of a given data is the difference between the maximum value and the minimum value of a variate.
7. ARRAY is an arrangement of raw numerical data in ascending or descending order.
8. GROUPED DATA is the data condensed into classes or groups.
9. CLASS INTERVAL is the group in which the raw data is condensed. It is bounded by the lower and the upper limits.
10. CLASS LIMITS are the two figures by which the class is bounded. The figure on the left is the **LOWER LIMIT** while the figure on the right is the **UPPER LIMIT**.
11. HISTOGRAM is a graphical representation of a frequency distribution in the form of rectangles with class intervals as bases and heights proportional to corresponding frequencies such that there is no gap between any two successive rectangles.

ALGORITHM FOR CONSTRUCTING A HISTOGRAM	
Step 1	On a graph paper draw X'X (x – axis) and YY' (y – axis) perpendicular to each other intersecting at O.
Step 2	Choosing a suitable scale represent the class intervals on the x – axis.
Step 3	Choosing a suitable scale mark the frequencies along the y – axis.
Step 4	Construct rectangles with class intervals as bases and respective frequencies as heights.

12. FREQUENCY is the number of times an observation occurs.
13. FREQUENCY POLYGON is a line graph of class frequency plotted against class midpoint.

ALGORITHM FOR CONSTRUCTING A FREQUENCY POLYGON	
Step 1	Construct a Histogram
Step 2	Choose the class intervals and mark the values on the x – axis.
Step 3	Mark the frequencies on the y – axis.
Step 4	Taking the mid – point of each class interval and the corresponding frequencies plot the points on the graph.
Step 5	Mark the midpoints of the class intervals preceding and succeeding the Histogram.
Step 6	Connect all the points with line segments.

14. PIE CHART is a pictorial representation of the numerical data by non – intersecting adjacent sectors of the circle such that each sector is proportional to the data represented by the sector.
15. ANGLE REPRESENTED BY A SECTOR = $\frac{\text{Value of a component}}{\text{Sum of all the components}} \times 360°$

ALGORITHM FOR CONSTRUCTING A PIE DIAGRAM	
Step 1	Obtain the data and find the sum of all components.
Step 2	Using the sector formula find the angle for each component.
Step 3	Draw a circle of any size and mark the centre of the circle.
Step 4	Draw a radius from the centre to the circumference.
Step 5	Taking the radius in Step 4 as the base construct an angle corresponding to a sector.
Step 6	Moving in clockwise direction, construct angles corresponding to other components taking the new arm of each angle as base.
Step 5	Label each sector as per its sector angle.

16. FREQUENCY DISTRIBUTION is the tabular arrangement of data showing the frequency distribution.
17. EXCLUSIVE (CONTINUOUS) FORM of frequency distribution is that in which the upper limit is excluded while the lower limit is included. Example : 10 – 20, 20 – 30, etc. **[lower limit ≤ x < upper limit]**
18. INCLUSIVE (DISCONTINUOUS) FORM of frequency distribution is that in which both the upper limit and the lower limit is included. Example : 11 – 20, 21 – 30, etc. **[lower limit ≤ x ≤ upper limit]**
19. TRUE LOWER AND UPPER LIMITS in an exclusive form are the limits (boundaries) by which the class is bounded.
20. TRUE LOWER AND UPPER LIMITS in an inclusive form are the limits (boundaries) obtained by adding 0.5 to the upper limit and subtracting 0.5 from the lower limit. Example : 11 – 20 will become 10.5 – 20.5.
21. CLASS SIZE is the difference between the true upper limit and the true lower limit of a class.

22. CLASS MARK is obtained by adding the upper limit and the lower limit and dividing the answer by 2.
23. CUMULATIVE FREQUENCY corresponding to a class is the sum of all frequencies up to and including that class.
24. MEAN is the average of a given set of numbers. It is represented as $\bar{x}$.
    (a) For ungrouped data $\bar{x} = \frac{\text{sum of all observations}}{\text{number of observations}}$
    (b) For grouped data :

DIRECT METHOD	SHORT CUT METHOD	STEP DEVIATION METHOD
$\bar{x} = \frac{\sum fx}{\sum f}$	$\bar{x} = A + \left[\frac{\sum fd}{\sum f}\right]$	$\bar{x} = A + \left[\frac{\sum f i}{\sum f}\right] \times h$
NOMENCLATURE : A = assumed mean,	d = x – A,	$i = \frac{d}{h}$, h = height of C. I

25. MEDIAN is the mid value of a set of numbers arranged in either ascending or descending order.

UNGROUPED DATA		GROUPED DATA
Median = $\left(\frac{n+1}{2}\right)^{th}$ term	Median = $\frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}\right]$	Median = $L + \frac{\frac{N}{2} - cf}{f} \times h$
When n is ODD	When n is EVEN	
NOMENCLATURE : L is lower limit of the median class N is the total of the frequencies, c f is the cumulative frequency before the median class, f is the frequency of the median class, h is the height of the class interval		

26. QUARTILES are values that divide a set of values into four equal parts. **Lower Quartile (Q_1)** is the 25th percentile of the given data and is $\left(\frac{n+1}{4}\right)^{th}$ term. **Upper Quartile (Q_3)** is the 75th percentile of the given data and is $\left(\frac{3(n+1)}{4}\right)^{th}$ term, where n is the total number of observations.
27. OGIVE is a graph that shows the distribution of cumulative frequency for a set of data. It is also called a cumulative frequency curve and is created by taking the class intervals on the x – axis and the frequencies on the y – axis.

USES OF OGIVE	
1	To find the median of a set of data.
2	To find the lower quartile and the upper quartile.
3	To calculate the percentage of a set of values.
4	To determine how many observations are less than or more than or equal to a specific value.
5	To identify the popularity of data.
6	To determine the likelihood of data falling within a certain frequency range.

ALGORITHM FOR CONSTRUCTING AN OGIVE	
Step 1	Create a frequency table from the given data.
Step 2	Find the cumulative frequency for each class interval
Step 3	Mark the class boundaries on the x – axis and the frequencies on the y – axis.
Step 4	Plot the points by taking the upper limit of a class interval and the corresponding cumulative frequency.
Step 5	Join the points freely to form LESS THAN or MORE THAN OGIVE.

28. MODE is the value that appears most often in a set of data values.
29. TO FIND MODE BY EMPERICAL FORMULA : **Mode = 3 median – 2 mean**
30. TO FIND MODE OF A GROUPED DATA : **Mode** = $L + \frac{f - f_0}{2f - f_0 - f_1} \times h$
    [L is the lower limit of the modal class, f is the frequency of the modal class, f_0 is the frequency of the class above the modal class, f_1 is the frequency of the class below the modal class, h is the height of the class interval]
31. MEAN DEVIATION is a statistical measure that computes the average deviation from the mean value of a given data.
    Mean Deviation = $\frac{\sum|x - \bar{x}|}{N}$ [x is the value in the data set, $\bar{x}$ is the mean of the given data set, N is the number of frequencies.]
32. VARIANCE is the mean squared difference between each data point and the mean. It is a measure of dispersion that takes into account the spread of all data points in a data set. It is denoted as σ^2.

UNGROUPED DATA	GROUPED DATA DIRECT METHOD	STEP DEVIATION METHOD
$\sigma^2 = \frac{\sum d^2}{N}$	$\sigma^2 = \frac{\sum f d^2}{N}$	$\sigma^2 = \frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2 \times h$
NOMENCLATURE : d is the mean deviation,	N is the total number of frequencies	h is the height of the C. I

ALGORITHM TO CALCULATE VARIANCE			
STEPS	UNGROUPED DATA	DISCRETE FREQUENCY DATA	GROUPED DATA
1	Find the mean of the given observations.	Multiply the variable given by the respective frequency and add to get Σ fx	Find the mid points of each class interval and let that be x
2	Subtract the mean obtained above from each observation to obtain d	Divide Σ fx by N to get the mean $\bar{x}$	Take the deviations of these midpoints from an assumed mean, A
3	Square each observation obtained above and add them to obtain Σ d^2	Subtract the mean from each observation $(x - \bar{x})$	Divide the deviations obtained above by the class interval h to get u
4	Divide Σd^2 by N (total number of observations to get the variance.	Square each of the observations obtained above $(x - \bar{x})^2$	Multiply the frequency of each class with the corresponding u to get fu
5		Multiply each squared deviation by their respective frequencies and add to get Σ $f(x - \bar{x})^2$	Square the values of u and multiply with their respective frequencies and add to obtain Σ fu^2.
6		Divide Σ $f(x - \bar{x})^2$ by N to get the variance.	Find the variance using the step deviation method.

33. COEFFICIENT OF VARIANCE is defined as a ratio of standard deviation to the mean. **CV =** $\frac{\sigma}{\bar{x}}$ **X 100**
34. STANDARD DEVIATION is a measure which shows how much variation from the mean exists. A low SD means that the numbers are clustered close to the mean and a high SD means the numbers are more spread out. It is denoted by σ

For ungrouped date $\sigma = \sqrt{\text{variance}} = \sqrt{\frac{\Sigma d^2}{N}}$	For grouped data $\sigma = \sqrt{\frac{\Sigma f d^2}{N}}$	Step Deviation method $\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$ x h

35. SKEWNESS is a measure of symmetry or distortion of symmetric distribution. It measures the deviation of the given distribution of a random variable from a symmetric distribution.
36. POSITIVE SKEWNESS is seen in a frequency curve when the tail on the right side of the curve is longer.
Mean > Median > Mode.
37. NEGATIVE SKEWNESS is seen in a frequency curve when the tail on the left side of the curve is longer.
Mean < Median < Mode.
38. ABSOLUTE MEASURES OF SKEWNESS tells us the extent of asymmetry and whether it is positive or negative.
39. COEFFICIENT OF SKEWNESS is a measure to determine the strength and direction of the skewness of a sample distribution.

RULES TO KNOW THE DEGREE OF SKEWNESS		
	DESCRIPTION	TYPE
1	When the skewness lies between – 0.5 to 0.5	Fairly Symmetrical
2	When the skewness lies between – 1 to – 0.5	Negatively Skewed
3	When the skewness lies between 0.5 to 1	Positively Skewed
4	When the skewness is less than – 1 or greater than 1	Highly Skewed

40. MOMENTS are specific quantitative measures of the shape of a frequency distribution. It measures **SKEWNESS** and **KURTOSIS**.
41. r[th] MOMENT ABOUT MEAN is $\mu_r = \frac{\Sigma f_i (x - \bar{x})^r}{n}$, r = 0, 1, 2, … [$\bar{x}$ is the mean of the given data and n = Σf_i]

NOTE THAT $\mu_0 = 1$, $\mu_1 = 0$ a μ_2 = variance.

42. MOMENT COEFFICIENTS OF SKEWNESS are β_1 and γ_1. $\beta_1 = \frac{{\mu_3}^2}{{\mu_2}^3}$ $\gamma_1 = \pm\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{{\mu_2}^3}}$
43. KARL PEARSON'S COEFFICIENT OF SKEWNESS is the ratio of the difference of mean and mode to the standard deviation. $S_{KP} = \frac{\text{Mean} - \text{Mode}}{\text{Standard Deviation}}$ OR $S_{KP} = \frac{3\,(\text{Mean} - \text{Median})}{\text{Standard Deviation}}$

- If $S_{KP} > 0$, then the distribution is positively skewed
- If $S_{KP} = 0$, then the distribution is symmetrical
- If $S_{KP} < 0$, then the distribution is negatively skewed.

44. BOWLEY'S COEFFICIENT OF SKEWNESS is the ratio of the difference of the upper and lower quartiles to the inter quartile range. $S_{KB} = \frac{Q_3 + Q_1 - 2\,\text{Median}}{Q_3 - Q_1}$
45. KURTOSIS is a statistical measure that describes how peaked or flat a distribution is compared to normal distribution.
46. POSITIVE KURTOSIS is a relatively peaked distribution with too few observations in the tail.
47. NEGATIVE KURTOSIS is a relatively flat distribution with too many observations in the tail.

48. LEPOKURTIC is a distribution with relatively large tails and a high risk of extreme returns. ($\beta_2 > 3$)
49. PLATYKURTIC is a distribution with relatively small tails and a very low risk of extreme returns. ($\beta_2 < 3$)
50. MESOKURTIC is a distribution with relatively medium tails and a moderate risk of extreme returns. ($\beta_2 = 3$)

NOTE : $\beta_2 = \frac{\mu_4}{{\mu_2}^2}$

	For Ungrouped Data	For Discontinuous Grouped Data	For Continuous Grouped Data
PERCENTILE RANK	$PR = \frac{L + 0.5\,E}{N} \times 100$	$PR = \frac{C + 0.5\,f}{N} \times 100$	$PR = \left(C + \frac{x}{h}(f)\right)\frac{100}{N}$

L is number of observations less than x_i, E is number of observations equal to x_i, N is total number of observations, C is the cumulative frequency above x_i, f is the frequency of x_i, h is the height of the class interval

51. CORRELATION is a statistical measure that shows how two variables change together at a constant rate.
    Example : The relationship between the height and weight of a person.
52. SIMPLE CORRELATION is defined as the amount of similarity in direction and degree of variations in corresponding pairs of observation of only two variables.
    Example : Relationship between use of fertilizers and yield of crop.
53. PARTIAL CORRELATION is when three or more variables are studied but only two are considered to be influencing each other.
54. MULTIPLE CORRELATION is when three or more variables are studied simultaneously.
55. POSITIVE CORRELATION means the variables move in the same direction.
56. NEGAITIVE CORRELATION means the variables move in opposite directions.
57. COVARIANCE of variables X and Y is the mean of the product of deviation scores.

    a) **Cov (X, Y) or** $C_{XY} = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{N}$ when $(x - \bar{x})$ and $(y - \bar{y})$ are small fraction less numbers.

    b) **Cov (X, Y) or** $C_{XY} = \frac{1}{N}\left[\Sigma xy - \frac{1}{N}\Sigma x\,\Sigma y\right]$ when x, y are small numbers.

    c) **Cov (X, Y) or** $C_{XY} = \frac{1}{N}\left[\Sigma uv - \frac{1}{N}\Sigma u\,\Sigma v\right]$ when $u = x - A$ and $v = y - B$

58. COEFFICIENT OF CORRELATION **r or ρ (x, y) =** $\frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(x)}\sqrt{\text{Var}(y)}}$
59. SCATTER DIAGRAMS are used to analyze relationship between variables. It uses dots to represent values for two different numeric values.

ALGORITHM FOR PLOTTING A SCATTER GRAPH	
Step 1	Collect pairs of data that have a relationship.
Step 2	Identify the dependent and independent variables. The independent variable remains constant while the dependent variable changes.
Step 3	On the x – axis mark the independent variable and on the y – axis mark the dependent variable. Include tick marks to cover all the values in the data.
Step 4	For each value of the date mark a dot or a symbol where the x – axis value crosses the y – axis value.
Step 5	Look for patterns to see a relationship.

TYPES OF CORRELATTION		
1	POSITIVE CORRELATION	The variables move in the same direction. Both will either be increasing or decreasing.
2	NEGATIVE CORRELATION	The variables move in opposite directions. One will be increasing while the other will be decreasing.
3	NULL CORRELATION	There is no clear correlation between the variables.

INTERPRETING A SCATTER DIAGRAM	
1	If the data points make a straight line going from the origin to high y values, then there is **POSITIVE CORRELATION**.
2	If the data points start at high y values and progress down to low values, then there is **NEGATIVE CORRELATION**.
3	If the points are clustered the relationship is strong.
4	If the points are spread out the relationship is weak.

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FORMULAE SHEET

CALCULUS

LIMITS

1. LIMITS is a concept that describes how a function behaves as its input approaches a certain value. They are used to assign values to functions at points where no values are defined.
2. RIGHT HAND LIMIT : A function possesses a Right Hand Limit as x approaches it from a value higher than the function. It is written as $x \to a^+$.
3. LEFT HAND LIMIT : A function possesses a Left Hand Limit as x approaches it from a value lower than the function. It is written as $x \to a^-$.

ALGORITHM		
	LEFT HAND LIMIT	RIGHT HAND LIMIT
STEP 1	Write $\lim_{x \to a^-} f(x)$	Write $\lim_{x \to a^+} f(x)$
STEP 2	Put x = a – h and replace $x \to a^-$ by $h \to 0$ to obtain $\lim_{h \to 0} f(a - h)$	Put x = a + h and replace $x \to a^+$ by $h \to 0$ to obtain $\lim_{h \to 0} f(a + h)$
STEP 3	Simplify $\lim_{h \to 0} f(a - h)$ by using the formula for the given function.	Simplify $\lim_{h \to 0} f(a + h)$ by using the formula for the given function.
STEP 4	The value obtained above is the LHL of $f(x)$ at x = a	The value obtained above is the RHL of $f(x)$ at x = a

4. ODD FUNCTION is that where the equation $-f(x) = f(-x)$ holds true for all values of x in its domain.
5. EVEN FUNCTION is that where the equation $f(x) = f(-x)$ holds true for all values of x in its domain.
6. EVALUATION OF ALGEBRAIC LIMITS is done by the following methods :
   (a) **Direct Substitution Method** : In this method as limit of $f(x)$ approaches a, we directly substitute the value a into the function to find the limit.
   (b) **Factorization Method** : In this method we factorize both the numerator and the denominator and cancel out the common factors. Thereafter we use the direct substitution method to obtain the limit.
   (c) **Rationalization Method** : In this method we multiply the numerator and the denominator by the conjugate of the denominator and evaluate the limit by direct substitution. We can have 0 in the numerator but can never have a 0 in the denominator.
   (d) **By using some Standard Limits given below**

STANDARD RESULTS OF LIMITS				
$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n\,a^{n-1}$	$\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}$	$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$	$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$	$\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$
$\lim_{x \to 0} \left(1 + \frac{1}{x}\right)^x = e$	$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$	$\lim_{x \to 0} \frac{\sin x}{x} = 1$	$\lim_{x \to 0} \frac{\tan x}{x} = 1$	$\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx} = \frac{m^2}{n^2}$

EXPONENTIAL AND LOGARITHMIC LIMITS					
1	$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \ldots$	4	$\log(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} \ldots$	7	$\text{Sin } x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots$
2	$a^x = 1 + x(\log_e a) + \frac{x^2}{2!}(\log_e a)^2 + \frac{x^3}{3!}(\log_e a)^3 + \ldots$	5	$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \ldots$	8	$\text{Cos } x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots$
3	$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \ldots$	6	$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} \ldots$	9	$\text{Tan } x = x + \frac{x^3}{3} + \frac{2}{15} x^5 + \ldots$

CONTINUITY

1. CONTINUOUS FUNCTION is that function which is continuous at each point of its domain.
2. CONTINUITY IN AN INTERVAL : A function $f(x)$ is said to be continuous in an open interval $]a, b[$ if it is continuous at each point of]a, b[.
3. TEST OF CONTINUITY : A function $f(x)$ is said to be continuous at x = a iff $f(x) = f(a) = \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$.
4. PROPERTIES OF CONTINUITY

1	Every constant function is continuous.	5	Every logarithmic function is continuous.
2	Every identity function is continuous.	6	Every modulus function is continuous.
3	Every polynomial function is continuous.	7	Every trigonometric function is continuous.
4	Every differentiable function is continuous.		

5. If $f(x)$ and $g(x)$ are two real functions continuous at x = a, then

1	$f(x) + g(x)$ is continuous at x = a.	4	$\alpha f(x)$ is continuous at x = a and α is a real number.
2	$f(x) - g(x)$ is continuous at x = a.	5	$\frac{f(x)}{g(x)}$ is continuous at x = a provided $g(x) \neq 0$
3	$f(x)\, g(x)$ is continuous at x = a	6	$\frac{1}{f(x)}$ is continuous at x = a provided $f(x) \neq 0$

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DIFFERENTIABILITY

1. DIFFERENTIABILITY of a function in an open interval (a, b) if the derivative of the function exists at all points in the domain. [If $c \in (a, b)$ then $f(x)$ is said to be differentiable at $x = c$ if and only if $\lim_{x=c} \frac{f(x) - f(c)}{x - c}$ exists finitely.
2. RIGHT HAND DERIVATIVE of $f(x)$ at x = a is $\lim_{h=0} \frac{f(a+h) - f(a)}{h}$ if it exists and is denoted by $Rf'(a)$.
3. LEFT HAND DERIVATIVE of $f(x)$ at x = a is $\lim_{h=0} \frac{f(a-h) - f(a)}{-h}$ if it exists and is denoted by $Lf'(a)$.

ALGORITHM TO KNOW THE DIFFERENTIABILITY OF A FUNCTION	
Step 1	Find the right hand derivative of the given function at the given point.
Step 2	Find the left hand derivative of the given function at the given point.
Step 3	The function $f(x)$ is differentiable at $x = a$ only if $Rf'(a) = Lf'(a)$
Step 4	The function $f(x)$ is not differentiable at $x = a$ only if $Rf'(a) \neq Lf'(a)$

USEFUL RESULTS ON DIFFERENTIABILITY	
1	Every polynomial function is differentiable at each $x \in R$.
2	The exponential function a^x, $a > 0$ is differentiable at each $x \in R$.
3	Every constant function is differentiable at each $x \in R$.
4	The logarithmic function is differentiable at each point in its domain.
5	Trigonometric and Inverse Trigonometric functions are differentiable in their respective domains.
6	The sum, difference, product and quotient of two differentiable functions is also differentiable.
7	The composition of a differentiable function is a differentiable function.

DERIVATIVES

1. DIFFERENTIATION is the process of finding the derivative of a function which measures the rate of change of that function with respect to its variables.
2. DERIVATIVE is the result of differentiation. It represents the rate of change or the slope of the function at any given point.
3. METHODS OF DIFFERENTIATION : The different methods used to differentiate a function are Power Rule, Product Rule, Quotient Rule, Chain Rule, through Exponential and Logarithmic functions, through Parametric functions, Implicit functions, etc.

NOTE
1. The derivative of an odd function is an even function while that of an even function is odd.
2. Every differential function is continuous but every continuous function need not be differentiable.
3. When a given function is a power of some expressions or a product of expressions then take log on both sides and differentiate term wise.

DERIVATIVES OF STANDARD FUNCTIONS					
1	$\frac{d}{dx}(x^n) = n\,x^{n-1}$	8	$\frac{d}{dx}(\sin x) = \cos x$	15	$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
2	$\frac{d}{dx}(k) = 0$	9	$\frac{d}{dx}(\cos x) = -\sin x$	16	$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
3	$\frac{d}{dx}(e^x) = e^x$	10	$\frac{d}{dx}(\tan x) = \sec^2 x$	17	$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
4	$\frac{d}{dx}(a^x) = a^x \log a$	11	$\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cot x$	18	$\frac{d}{dx}(\mathrm{cosec}^{-1} x) = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$
5	$\frac{d}{dx}(\log x) = \frac{1}{x}$	12	$\frac{d}{dx}(\sec x) = \sec x \tan x$	19	$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
6	$\frac{d}{dx}(\log_a x) = \frac{1}{x \log a}$	13	$\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$	20	$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
7	$\frac{d}{dx}(ax^n) = a\,n\,x^{n-1}$	14	$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$	21	$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

NOTE
$\frac{d}{dx}$ is an operator such that when it is applied on $y = f(x)$ gives us $\frac{d}{dx}(f(x)) = \frac{dy}{dx}$.
$\frac{dy}{dx}$ is not simply a fraction obtained by dividing dy by dx. For example $\frac{d}{dx}$ when applied on $\sin x$ gives us $\cos x$.

4. RULES OF DIFFERENTIATION

1	Constant Function Rule	If $f(x) = A$ then $f'(x) = 0$ where A is a constant.
2	Linear Function Rule	If $f(x) = a + bx$ then $f'(x) = b$ where a and b are constants.
3	Power Function Rule	If $f(x) = x^n$ then $f'(x) = n\,x^{n-1}$
4	Sum and Difference Rule	If $u = f(x)$ and $v = g(x)$, then $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$ For multiple functions $\frac{d}{dx}(u \pm v \pm w \pm \ldots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} + \ldots$
5	Product Rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
6	Quotient Rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$
7	Chain Rule	If $z = f(u)$, $u = f(v)$ and $v = f(x)$, then $\frac{dz}{dx} = \frac{dz}{du} \times \frac{du}{dv} \times \frac{dv}{dx}$
8	Inverse Function Rule	If $y = f^{-1}(x)$ then $\frac{dy}{dx} = \frac{1}{f'(f^{-1}(x))}$

5. DIFFERENTIATION BY FIRST PRINCIPLE : $\frac{d}{dx}\{f(x)\} = \lim_{h \to 0} \frac{f(x \pm h) - f(x)}{\pm h}$.

6. DIFFERENTIATION BY TRIGONOMETRIC TRANSFORMATIONS

USEFUL RESULTS							
1	$1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)$	3	$\sin x = \frac{2\tan(x/2)}{1 + \tan^2(x/2)}$	5	$\sin 3x = 3\sin x - 4\sin^3 x$	7	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$
2	$1 + \cos x = 2\cos^2\left(\frac{x}{2}\right)$	4	$\cos x = \frac{1 - \tan^2(x/2)}{1 + \tan^2(x/2)}$	6	$\cos 3x = 4\cos^3 x - 3\cos x$	8	$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

USEFUL SUBSTITUTIONS	
Given $\sin^{-1} f(x)$, $\cos^{-1} f(x)$, $\tan^{-1} f(x)$, etc.	
Rule 1	If $f(x) = \sqrt{a^2 - x^2}$, put $x = a\sin\theta$ or $x = a\cos\theta$
Rule 2	If $f(x) = \sqrt{a^2 + x^2}$, put $x = a\tan\theta$ or $x = a\cot\theta$
Rule 3	If $f(x) = \sqrt{x^2 - a^2}$, put $x = a\sec\theta$ or $x = a\,\mathrm{cosec}\,\theta$
Rule 4	If $f(x) = \sqrt{a - x}$, put $x = a\cos 2\theta$

7. **ROLLES THEOREM** states that if a function $f(x)$ is continuous on the closed interval [a, b], is differentiable in the open interval (a, b) and if $f(a) = f(b)$, then there exists at least one number c in (a, b) such that $f'(c) = 0$

8. **LAGRANGE'S MEAN VALUE THEOREM** states that if a function $f(x)$ is continuous on the closed interval [a, b], is differentiable in the open interval (a, b), then there exists a point c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

REMEMBER	1. A **Polynomial** function is everywhere continuous and differentiable. 2. The **Exponential** function, **Sine** and **Cosine** functions are everywhere continuous and differentiable. 3. A **Logarithmic** function is continuous and differentiable in its domain. 4. **Tan x** is not continuous at $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$ 5. $\lvert x \rvert$ is not differentiable at $x = 0$ 6. If $f'(x)$ tends to ∞ as $x \to k$, then $f'(x)$ is not differentiable at $x = k$. 7. The sum, difference, product and quotient of a continuous (differential) function is also continuous (differentiable).

9. **L'HOSPITAL'S RULE** states that if $f(x)$ and $g(x)$ are differentiable functions such that $g'(x) \neq 0$ near a point a and if $\frac{f(x)}{g(x)}$ is an indeterminate form then the limit as x approaches a of $\frac{f(x)}{g(x)}$ is equal to the limit as x approaches a of $\frac{f'(x)}{g'(x)}$.

10. **IMPLICIT FUNCTION** is a function $f(x, y) = a$ in which x and y are defined in such a way that y is not expressible directly in terms of x.

ALGORITHM TO DIFFERENTIATE AN IMPLICIT FUNCTION	
Step 1	Differentiate both sides of the equation with respect to the independent variable.
Step 2	Apply the chain rule involving the dependent variable.
Step 3	Simplify the equation.
Step 4	Collect all terms involving the derivative $\frac{dy}{dx}$ on one side of the equation.
Step 5	Solve for $\frac{dy}{dx}$

11. **PARAMETRIC DIFFERENTIATION** is a technique for finding the derivative of a function when the function's variables are related to a third variable.

ALGORITHM FOR PARAMETRIC DIFFERENTIATION	
Step 1	Write the functions x and y in terms of the parameter t.
Step 2	Find the derivatives $\frac{dy}{dt}$ and $\frac{dx}{dt}$.
Step 3	Use the formula $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$
Step 4	Substitute the values of $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and simplify.

12. **ERROR** is the difference between the true value and the observed or measured value.
13. **ABSOLUTE ERROR** is the absolute value of the difference between the true value and the measured value. It is denoted by Δx.
14. **RELATIVE ERROR** is a way of measuring the difference between the true value and the measured value and is expressed as a ratio of the absolute error to the actual. It is denoted by $\frac{\Delta x}{x}$.
15. **PERCENTAGE ERROR** is $\frac{\Delta x}{x} \times 100$.
16. **EQUATION OF A TANGENT TO A CURVE** is $y - y_1 = \frac{dy}{dx}(x - x_1)$, where (x_1, y_1) is a point and $\frac{dy}{dx}$ is a slope of the tangent.
17. **EQUATION OF A NORMAL TO A CURVE** is $y - y_1 = \frac{-1}{\frac{dy}{dx}}(x - x_1)$

ALGORITHM TO FIND THE EQUATION OF TANGENT AND NORMAL	
Step 1	Find $\frac{dy}{dx}$ from the given function $y = f(x)$
Step 2	Find the value of $\frac{dy}{dx}$ at the given point (x_1, y_1)
Step 3	If the value of $\frac{dy}{dx}$ at the point (x_1, y_1) is non – zero finite number, then obtain the equation of the tangent and the normal by using the formulae given above in No. 16 and No. 17.

18. **ORTHOGONAL CIRCLES (CURVES)** are those 2 circles (curves) whose tangents at the point of intersection of the circles (curves) are perpendicular to each other.
19. **CONDITION FOR ORTHOGONALITY OF CIRCLES (CURVES)** is $\left(\frac{dy}{dx}\right)_{c_1} \times \left(\frac{dy}{dx}\right)_{c_2} = -1$.
20. **ANGLE OF INTERSECTION OF 2 CIRCLES (CURVES)** is $\tan\theta = \frac{\left(\frac{dy}{dx}\right)_{c_1} - \left(\frac{dy}{dx}\right)_{c_2}}{1 + \left(\frac{dy}{dx}\right)_{c_1}\left(\frac{dy}{dx}\right)_{c_2}}$

NOTE
If $\frac{dy}{dx} = \propto$, then the tangent is parallel to the y – axis and if $\frac{dy}{dx} = 0$ then the tangent is parallel to the x – axis.

21. **MONOTONIC FUNCTION** is a function in a closed interval [a, b] that is either increasing or decreasing in the open interval (a, b).
22. **INCREASING FUNCTION** is a function where the value of $f(x)$ increases as x increases at all points in the domain. For all $x < y$, we have $f(x) \leq f(y)$
23. **STRICTLY INCREASING FUNCTION** is a function where the value of $f(x)$ is always increasing over its domain as x increases. For all $x < y$, we have $f(x) < f(y)$.
24. **DECREASING FUNCTION** is a function where the value of $f(x)$ decreases as x increases at all points in the domain. For all $x < y$, we have $f(x) \geq f(y)$.
25. **STRICTLY DECREASING FUNCTION** is a function where the value of $f(x)$ is always decreasing over its domain as x increases. For all $x < y$, we have $f(x) > f(y)$.
26. **ABSOLUTE MAXIMA** is a point in the domain where the function $f(x)$ obtains its greatest possible value.
27. **ABSOLUTE MINIMA** is a point in the domain where the function $f(x)$ obtains its smallest possible value.

ALGORITHM TO FIND THE INTERVAL IN WHICH A FUNCTION IS INCREASING OR DECREASING	
Step 1	Obtain the function and put it equal to $f(x)$.
Step 2	Find $\frac{dy}{dx}$
Step 3	Put $\frac{dy}{dx} > 0$ and solve the inequation.
Step 4	For the values of x obtained in step 3, $\frac{dy}{dx}$ is increasing and for the remaining points in the domain it is decreasing.

28. L**OCAL MAXIMA** is a point where the values of $f(x)$ near that point are always less than the value of the function at that point. $f(x) < f(a)$ for all $x \in (a - \delta, a + \delta), x \neq a$
29. **LOCAL MINIMA** is a point where the values of $f(x)$ near that point are always greater than the value of the function at that point. $f(x) > f(a)$ for all $x \in (a - \delta, a + \delta), x \neq a$

30. **POINT OF LOCAL MAXIMUM VALUE** is that point where x = a and $f'(a) = 0$ and $\frac{dy}{dx}$ changes from positive to negative as x increases through a.
31. **POINT OF LOCAL MANIMUM VALUE** is that point where x = a and $f'(a) = 0$ and $\frac{dy}{dx}$ changes from negative to positive as x increases through a.
32. **EXTREMUM VALUES** of a function are the points at which the function's maximum or minimum values occur in a given interval. There are 2 types of extrema, Absolute and Relative.
33. **STATIONARY POINT** of a function $f(x)$ is that point where $\frac{dy}{dx} = 0$ and the function is neither increasing nor decreasing.

FIRST DERIVATIVE TEST FOR LOCAL MAXIMA AND MINIMA	
Step 1	Put the function as $y = f(x)$
Step 2	Find $\frac{dy}{dx}$
Step 3	Put $\frac{dy}{dx} = 0$ and solve the equation to obtain the values of x. Let the roots be a, b, c, … These points are to be considered for finding the local maxima and local minima.
Step 4	Test the function at each one of these values. Suppose x = c
Step 5	Determine the sign of $\frac{dy}{dx}$ for values of x slightly less than c and slightly greater than c.
Step 6	If $\frac{dy}{dx}$ changes its sign from positive to negative as x increases through c, then x = c is a point of local maxima.
Step 7	If $\frac{dy}{dx}$ changes its sign from negative to positive as x increases through c, then x = c is a point of local minima.
Step 8	If $\frac{dy}{dx}$ does not change sign as x increases through c, then x = c is called a point of inflexion.

ALGORITHM FOR THE HIGHER ORDER DERIVATIVE	
Step 1	Put the function as $y = f(x)$ and find $\frac{dy}{dx}$
Step 2	Put $\frac{dy}{dx} = 0$ and solve the equation for x. Let the roots be a, b, c, etc. The roots of this equation are the stationary values of x and are the possible points where the function $f(x)$ can attain a local maxima or a local minima.
Step 3	Test the function at each of these roots.
Step 4	Find $\frac{d^2y}{dx^2}$
Step 5	For x = c find the value of $\frac{d^2y}{dx^2}$
Step 6	If $\frac{d^2y}{dx^2}$ at c < 0, then x = c is a point of local maximum.
Step 7	If $\frac{d^2y}{dx^2}$ at c > 0, then x = c is a point of local minimum.
Step 8	If $\frac{d^2y}{dx^2}$ at c = 0, then find $\frac{d^3y}{dx^3}$ and substitute in it c for x.
Step 9	If $\frac{d^2y}{dx^2} \neq 0$, then x = c is the point of inflexion.
Step 10	If $\frac{d^3y}{dx^3} = 0$, then find $\frac{d^4y}{dx^4}$ and substitute in it c for x.
Step 11	If $\frac{d^4y}{dx^4}$ at c < 0, then x = c is a point of local maximum.
Step 12	If $\frac{d^4y}{dx^4}$ at c > 0, then x = c is a point of local minimum.

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INTEGRATION

INDEDFINITE INTEGRALS OF STANDARD FUNCTIONS

1	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$	16	$\int \sin x\,dx = -\cos x + C$	31	$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1}x + C$
2	$\int \frac{1}{x}\,dx = \log\lvert x\rvert + C$	17	$\int \cos x\,dx = \sin x + C$	32	$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C$
3	$\int 1\,dx = x + C$	18	$\int \tan x\,dx = -\log\lvert\cos x\rvert + C$	33	$\int \frac{-1}{1+x^2}\,dx = \cot^{-1}x + C$
4	$\int x\,dx = x^2 + C$	19	$\int \cot x\,dx = \log\lvert\sin x\rvert + C$	34	$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + C$
5	$\int k\,dx = kx + C$	20	$\int \sec x\,dx = \log\lvert\sec x + \tan x\rvert + C$	35	$\int \frac{-1}{x\sqrt{x^2-1}}\,dx = \operatorname{cosec}^{-1}x + C$
6	$\int e^x\,dx = e^x + C$	21	$\int \operatorname{cosec} x\,dx = \log\lvert\operatorname{cosec} x - \cot x\rvert + C$	36	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
7	$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$	22	$\int \sec^2 x\,dx = \tan x + C$	37	$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + C$
8	$\int a^x\,dx = \frac{a^x}{\log_e x} + C$	23	$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$	38	$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
9	$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + c$	24	$\int \sec x \tan x\,dx = \sec x + C$	39	$\int \frac{-1}{a^2+x^2}\,dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$
10	$\int (ax+b)^n\,dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + C$	25	$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$	40	$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
11	$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log(ax+b) + C$	26	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$	41	$\int \frac{-1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\operatorname{cosec}^{-1}\left(\frac{x}{a}\right) + C$
12	$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$	27	$\int \sin(ax+b)\,dx = \frac{-1}{a}\cos(ax+b) + C$	42	$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$
13	$\int a^{bx+c}\,dx = \frac{1}{b}\frac{a^{bx+c}}{\log a} + C$	28	$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$	43	$\int \operatorname{cosec}^2(ax+b)\,dx = \frac{-1}{a}\cot(ax+b) + C$
14	$\int \tan(ax+b)\,dx = \frac{-1}{a}\log\lvert\cos(ax+b)\rvert + C$	29	$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left\lvert\frac{a+x}{a-x}\right\rvert + C$	44	$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left\lvert x + \sqrt{x^2-a^2}\right\rvert + C$
15	$\int \cot(ax+b)\,dx = \frac{1}{a}\log\lvert\sin(ax+b)\rvert + C$	30	$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left\lvert\frac{x-a}{x+a}\right\rvert + C$	45	$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left\lvert x + \sqrt{x^2+a^2}\right\rvert + C$
46	$\int \sec(ax+b)\,dx = \frac{1}{a}\log\lvert\sec(ax+b) + \tan(ax+b)\rvert + C$				
47	$\int \operatorname{cosec}(ax+b)\,dx = \frac{1}{a}\log\lvert\operatorname{cosec}(ax+b) - \cot(ax+b)\rvert + C$				
48	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\sin^{-1}\left(\frac{x}{a}\right)\right] + C$				
49	$\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2-a^2} + a^2\log\left(x+\sqrt{x^2-a^2}\right)\right] + C$				
50	$\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2+a^2} + a^2\log\left(x+\sqrt{x^2+a^2}\right)\right] + C$				

ALGORITHM FOR INTEGRALS OF THE FORM $\int \frac{P(x)}{(ax+b)^n}\,dx$, WHERE $P(x)$ IS A POLYNOMIAL

Step 1	Check whether degree of the numerator is greater than or equal to or less than the degree of the denominator.
Step 2	If degree of the numerator is greater than or equal to the degree of the denominator, divide the numerator by the denominator to express $\frac{P(x)}{(ax+b)^n}$ as $Q(x) + \frac{R(x)}{(ax+b)^n}$, where the degree of R(x) is less than n.
Step 3	If degree of the numerator is less than the degree of the denominator then expand the polynomial in the numerator.
Step 4	Thus $\int \frac{P(x)}{(ax+b)^n}\,dx = \int\left(\frac{A}{(ax+b)^n} + \frac{B}{(ax+b)^{n-1}} + \frac{C}{(ax+b)^{n-2}} + \cdots\right)dx$
Step 5	Now integrate RHS of Step 4 separately using basic integral rules.

ALGORITHM FOR INTEGRALS OF THE FORM $\int \frac{f'(x)}{f(x)}\,dx$

Step 1	Let the integral be $I = \int \frac{f'(x)}{f(x)}\,dx$
Step 2	Put $f(x) = t$ and replace $f'(x)\,dx$ by dt to get $I = \int \frac{1}{t}\,dt$
Step 3	Evaluate the integral obtained in Step 2 to obtain $I = \log\lvert t\rvert + c$
Step 4	Replace t by $f(x)$ in Step 3 to obtain $I = \log\lvert f(x)\rvert + c$.

ALGORITHM FOR INTEGRALS OF THE FORM $\int (ax+b)^n P(x)\,dx$ OR $\int \frac{P(x)}{(ax+b)^n}\,dx$, WHERE P (x) IS A POLYNOMIAL

Step 1	Substitute $(ax + b) = t$.
Step 2	Differentiate both sides to $dx = \frac{1}{a}\,dt$.
Step 3	Simplify the integrand in terms of t and integrate with respect to t by using $\int t^n\,dt = \frac{t^{n+1}}{n+1} + C$
Step 4	Replace t by $(ax + b)$ in Step 3.

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METHODS OF INTEGRATION

1. INTEGRATION BY SUBSTITUTION
   a) If the integrand is of the form f (ax + b), put (ax + b) = t and differentiate to get dx = $\frac{1}{a}$ dt.
   b) If the integrand is of the form $x^{n-1} f(x^n)$, put (x^n) = t and differentiate to get nx^{n-1} dx = dt.
   c) If the integrand is of the form $[f(x)]^n . f'(x)$, put f(x) = t and differentiate to get f'(x)dx = dt
   d) If the integrand is of the form $\frac{f'(x)}{f(x)}$, put f (x) = t and differentiate to get f'(x)dx = dt

ALGORITHM FOR INTEGRATION BY SUBTITIUTION	
Step 1	Identify the integral in the form $\int f\left(g(x)\right)g'(x)dx$
Step 2	Substitute g(x) as some constant t.
Step 3	Differentiate the assumed function with respect to t.
Step 4	Substitute g'(x) dx with dt.
Step 5	The resulting integral becomes $\int f(t)dt$
Step 6	Solve the integral using basic integration rules.
Step 7	Substitute g(x) back in for t.

2. INTEGRATION USING TRIGONOMETRIC IDENTITIES

USEFUL SUBSTITUTIONS			
INTEGRAND	**SUBSTITUTION**	**INTEGRAND**	**SUBSTITUTION**
$\int \sqrt{a^2 - x^2}\, dx$	Put x = a sin θ or x = a cos θ	$\int \sqrt{\frac{x}{a+x}}\, dx$	Put x = a $\tan^2 \theta$
$\int \sqrt{a^2 + x^2}\, dx$	Put x = a tan θ or x = a cot θ	$\int \sqrt{\frac{x}{a-x}}\, dx$	Put x = a $\sin^2 \theta$
$\int \sqrt{x^2 - a^2}\, dx$	Put x = a sec θ or x = a cosec θ	$\int \sqrt{a \sin x + b \cos x}\, dx$	Put a = r cos θ and b = r sin θ
$\int \sqrt{\frac{a-x}{a+x}}\, dx$ or $\int \sqrt{\frac{a+x}{a-x}}\, dx$	Put x = a cos 2 θ	$\int \sqrt{(x-a)(x-b)}\, dx$	Put x = a $\sec^2 \theta$ – b $\tan^2 \theta$
$\int \sqrt{\frac{a^2-x^2}{a^2+x^2}}\, dx$ or $\int \sqrt{\frac{a^2+x^2}{a^2-x^2}}\, dx$	Put $x^2 = a^2 \cos^2 2\theta$	$\int \sqrt{(x-a)(b-x)}\, dx$	Put x = a $\cos^2 \theta$ + b $\sin^2 \theta$
$\int \sqrt{\frac{2x}{1+x^2}}\, dx$ or $\int \sqrt{\frac{2x}{1+x^2}}\, dx$	Put x = tan θ	$\int \sqrt{ax - x^2}\, dx$	Put x = a $\sin^2 \theta$

ALGORITHM FOR INTEGRALS OF THE FORM $\int \frac{p(x)+q}{ax^2+bx+c} dx$	
Step 1	Write the numerator p (x) + q = α $\{\frac{d}{dx}(ax^2 + bx + c)\}$ + β i.e. p(x) + q = α (2ax + b) + β.
Step 2	Comparing LHS and RHS obtain the values of α and β.
Step 3	Replace p (x) + q by α (2ax + b) + β in the integral to get $\int \frac{p(x)+q}{ax^2+bx+c} dx = \alpha \int \frac{2ax+b}{ax^2+bx+c} dx + \beta \int \frac{1}{ax^2+bx+c} dx + C$.
Step 4	Integrate RHS in Step 3 and put the values of α and β obtained in Step 2.

ALGORITHM FOR INTEGRALS OF THE FORM $\int \frac{a \sin x + b \cos x}{c \sin x + d \cos x} dx$	
Step 1	Write the numerator (a sin x + b cos x) = α $\{\frac{d}{dx}(c \sin x + d \cos x)\}$ + β (c sin x + d cos x).
Step 2	Comparing LHS and RHS obtain the values of α and β.
Step 3	Replace the numerator (a sin x + b cos x) in the integrand by α (c cos x – d sin x) + β (c sin x + d cos x) to get $\int \frac{a \sin x + b \cos x}{c \sin x + d \cos x} dx = \alpha \int \frac{c \cos x - d \sin x}{c \sin x + d \cos x} dx + \beta \int \frac{c \sin x + d \cos x}{c \sin x + d \cos x} dx = \alpha \log \lvert c \sin x + d \cos x \rvert + \beta x + C$.
Step 4	Put the values of α and β obtained in Step 2.

3. INTEGRATION BY PARTS :
   If u and v are two functions of x then $\int (uv)\, dx = [u \int v\, dx] - \int \left\{\frac{du}{dx} . \int v\, dx\right\} dx$

REMEMBER ILATE RULE TO SELECT THE 1ST AND 2ND FUNCTIONS

Inverse trigonometric functions like $\sin^{-1} x$, $\cos^{-1} x$, etc.
Logarithmic function like log x, log x^2, etc.
Algebraic functions like x^2, x^3, $\sqrt{x}$, etc.
Trigonometric functions like sin x, cos x, etc.
Exponential functions like e^x, a^x, etc.

ALGORITHM FOR INTEGRATION BY PARTS	
Step 1	Choose the functions u and v.
Step 2	Differentiate u to get $\frac{du}{dx}$
Step 3	Integrate v to get $\int v\, dx$
Step 4	Put u, $\frac{du}{dx}$, $\int v$ dx in the formula $\int(uv)\,dx = [u \int v\,dx] - \int\left\{\frac{du}{dx} \cdot \int v\,dx\right\} dx$.
Step 5	Simplify and solve.

ALGORITHM FOR INTEGRALS OF THE FORM $\int e^x \{f(x) + f'(x)\}\,dx$	
Step 1	Express the integral as a sum of two integrals $f(x)$ and $f'(x)$, i.e. $\int e^x \{f(x) + f'(x)\}\,dx = \int e^x f(x)\,dx + \int e^x f'(x)\,dx$.
Step 2	Evaluate the first integral on RHS using integration by parts and taking e^x as the second part.
Step 3	The second integral on RHS will cancel out from the second term obtained by evaluating the first integral.
Step 4	Hence $\int e^x \{f(x) + f'(x)\}\,dx = e^x f(x) + C$.

4. BY PARTIAL FRACTIONS

DENOMINATOR	PARTIAL FRACTION
(x – a)	$\frac{A}{x-a}$
$(x-a)^2$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$(x-a)^3$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3}$
$(ax^2 + bx + c)$	$\frac{Ax+B}{ax^2+bx+c}$

NOTE

When integrating a rational algebraic function the degree of the numerator has to be always less than the degree of the denominator.

If the degree of the numerator is greater than or equal to the degree of the denominator then divide the numerator by the denominator to get a rational algebraic function where the degree of the numerator is less than the degree of the denominator.

NOTE

1. If the integral is of the form $\int \frac{dx}{ax^2 + bx + c}$ then by the method of completing a square, write the denominator in the form $[(x + m)^2 \pm n^2]$ and then integrate.
2. If the integral is of the form $\int \frac{px + q}{ax^2 + bx + c}$ dx, then write $(px + q) = A\frac{d}{dx}(ax^2 + bx + c) + B$. By comparison find A and B and then integrate.
3. If the integral is of the form $\int \frac{dx}{a + b\cos^2 x}$ or $\int \frac{dx}{a + b\sin^2 x}$ or $\int \frac{dx}{a\cos^2 x + b\cos x \sin x + c\sin^2 x}$ then divide both numerator and denominator by $\cos^2 x$ and put $\tan x = t$, $\sec^2 x\,dx = dt$ and then integrate.
4. If the integral is of the form $\int \sqrt{ax^2 + bx + c}$ dx, then write $(ax^2 + bx + c)$ as a $[(x + m)^2 \pm n^2]$ and then integrate.
5. If the integral is of the form $\int (px + q)\sqrt{ax2 + bx + c}$ dx, then write $(px + q) = A\frac{d}{dx}(ax^2 + bx + c) + B$. By comparison find A and B and then integrate.
6. If the integral is of the form $\int(\sin^m x \cos^n x)$ dx then check the exponents of sin x and cos x.
   a) If the exponent of sin x is an odd positive integer, then put cos x = t.
   b) If the exponent of cos x is an odd positive integer, then put sin x = t.
   c) If the exponents of both are odd positive integers, then put cos x = t or sin x = t.
   d) If the exponents of both are even positive integers, then put express $\sin^m x \cos^n x$ in terms of sines and cosines of multiples of x by using trigonometric results.
7. Evaluate the integral obtained above in 6.

ALGORITHM FOR INTEGRATING BY USING PARTIAL FRACTIONS	
Step 1	Factor the denominator into linear and quadratic factors.
Step 2	Write the rational fraction as a sum of simpler fractions.
Step 3	Find the values of the constants A, B and C either by comparison or by substitution.
Step 4	Integrate each partial fraction using substitiution.

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DEFINITE INTEGRALS

1. Definite integral is defined as the area under a curve between two fixed limits normally denoted by 'a' and 'b'. The numbers 'a' and 'b' are called the limits of integration. 'a' is called the lower limit and 'b' is called the upper limit.
2. $\int_a^b f(x)\,dx$ = [F(x) + C] – [F(a) + C] = F(b) – F(a)

ALGORITHM TO EVALUATE DEFINITE INTEGRAL	
Step 1	Find the indefinite integral $\int f(x)\,dx$. Let this be F(x). No need to write the constants of integration.
Step 2	Find the value of F(b) and F(a)
Step 3	Calculate F(b) – F(a)
	The value obtained in Step 3 is the value of the definite integral $\int_a^b f(x)\,dx$

ALGORITHM TO EVALUATE DEFINITE INTEGRAL BY SUBSTITUTION	
Step 1	Obtain the definite integral I = $\int_a^b f[\,g(x)\,g'(x)\,dx$
Step 2	Put $g(x)$ = t and get $g'(x)\,dx$ = dt
Step 3	Put x = a in t = $g(x)$ to get the new lower limit.
Step 4	Put x = b in t = $g(x)$ to get the new upper limit.
Step 5	Substitute $g'(x)\,dx$ = dt and replace old limits of integration to get $\int_{g(a)}^{g(b)} f(t)\,dt$
Step 6	Evaluate the integrand obtained in Step 5 by using standard methods of integration.

3. **PROPERTIES OF DEFINITE INTEGRALS**

1	Integration is independent of the change of variable	$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$
2	If the limits of a definite integral are interchanged then the value becomes negative.	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
3	If c lies between the limits [a, b], then	$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
4	If f(x) is a continuous function defined on [0, a] then	$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$
5	If f(x) is a continuous function defined on [a, b] then	$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$
6	The integrand of the sum of two functions	$\int_a^b [f(x) + \int_a^b g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)dx$
7	If f(x) is a continuous function defined on [– a, a] then	$\int_{-a}^a f(x)\,dx = \begin{cases} 0, & \text{when } f(x) \text{ is an odd function} \\ 2\int_0^a f(x)\,dx, & \text{when } f(x) \text{ is an even function} \end{cases}$
8	If f(x) is a continuous function defined on [0, 2a] then	$\int_0^{2a} f(x)\,dx = \begin{cases} 0, & \text{if } f(2a-x) = -f(x) \\ 2\int_0^a f(x)\,dx, & \text{if } f(2a-x) = f(x) \end{cases}$

4. **DEFINITE INTEGRAL AS THE LIMIT OF A SUM**

If f(x) is a continuous real valued function defined in the closed interval [a, b], then

$\int_a^b f(x)\,dx = \lim_{h\to 0} h\,[f(a) + f(a+h) + f(a+2h) + \dots + f[a+(n-1)h]]$ where nh = b – a

SOME USEFUL RESULTS FOR DIRECT APPLICATIONS			
1	$1 + 2 + 3 + \dots + (n-1) = \frac{n(n-1)}{2}$	3	$1^3 + 2^3 + 3^3 + \dots + (n-1)^3 = \left(\frac{n(n-1)}{2}\right)^2$
2	$1^2 + 2^2 + 3^2 + \dots + (n-1)^2 = \frac{n(n-1)(2n-1)}{6}$	4	$a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(r^n-1)}{r-1}$
5	$\sin a + \sin(a+h) + \sin(a+2h) + \dots + \sin[a+(n-1)h] = \frac{\sin\left\{a + \left(\frac{n-1}{2}\right)h\right\}\sin\left(\frac{nh}{2}\right)}{\sin\left(\frac{h}{2}\right)}$		
6	$\cos a + \cos(a+h) + \cos(a+2h) + \dots + \cos[a+(n-1)h] = \frac{\cos\left\{a + \left(\frac{n-1}{2}\right)h\right\}\sin\left(\frac{nh}{2}\right)}{\sin\left(\frac{h}{2}\right)}$		

ALGORITHM FOR DEFINITE INTEGRAL AS THE LIMIT OF A SUM	
Step 1	Divide the interval of integrations [a, b] into n sub intervals.
Step 2	Find the width of each rectangle, $\mathbf{h} = \frac{b-a}{n}$
Step 3	Find the height of each rectangle, $f(x_i)$.
Step 4	Find the sum of the areas of the rectangles , $S = \sum_{i=0}^{n} h\,f(x_i)$
Step 5	Take the limit as n approaches infinity, $\lim_{n\to\infty} S$

AREA OF BOUNDED REGIONS

1. AREA OF CURVE BETWEEN TWO ORDINATES
   If y = f(x) is a continuous and finite function in the closed interval [a, b] and if the curve lies above the x – axis, then the area bounded by the curve y = f(x) and the ordinates x = a and x = b is **Area =** $\int_a^b (y)\,dx$.
2. AREA OF CURVE BETWEEN TWO ABSCISSAE
   If y = f(x) is a continuous and finite function in the closed interval [a, b] and if the curve lies below the x – axis, then the area bounded by the curve y = f(x) and the ordinates x = a and x = b is **Area =** $\int_a^b (-y)\,dx$.
3. AREA BETWEEN TWO CURVES
   If y = f(x) is a continuous and finite function in the closed interval [a, b], then the area bounded by the curves y = f(x) and y = g(x) which are intersected by the ordinates x = a and x = b is **Area =** $\int_a^b \{f(x) - g(x)\}\,dx$.

ALGORITHM TO FIND THE AREA OF BOUNDED REGIONS	
Step 1	Make a rough sketch showing the area to be found.
Step 2	Slice the area into horizontal or vertical strips as the case maybe
Step 3	Consider a representative strip and the corresponding approximating rectangle.
Step 4	If the representative strip is parallel to the y – axis then its width is taken as Δ x.
Step 5	If the representative strip is parallel to the x – axis then its width is taken as Δ y.
Step 6	Find the area of the approximating rectangle.

ALGORITHM TO FIND THE AREA USING VERTICAL STRIPS	
Step 1	Make a rough sketch of the curve identifying the area to be found.
Step 2	Slice the area into vertical strips. Take an arbitrary point P (x, y) on the curve and construct a representative strip of width dx having two ends of its base on the x – axis at points (x – $\frac{dx}{2}$, 0) and (x + $\frac{dx}{2}$, 0) and (x, 0) as the midpoint of the base.
Step 3	Consider an approximate rectangle whose base is same as that of the representative strip and height equal to $\|y\| = \|f(x)\|$.
Step 4	Find the area of the approximating rectangle as $\|y\|$ dx = $\|f(x)\|$ dx.
Step 5	Find the value of x say $x = a$ and $x = b$ within which the approximating rectangle can move horizontally in the given region and form the integral $\int_a^b \|y\|$ dx.
Step 6	Evaluate the integral obtained in Step 5. The value of the integral obtained is the required area.

ALGORITHM TO FIND THE AREA USING HORIZONTAL STRIPS	
Step 1	Make a rough sketch of the curve identifying the area to be found.
Step 2	Slice the area into horizontal strips. Take an arbitrary point P(x, y) on the curve and construct a representative strip of width dy having (y, 0) as the midpoint of the base.
Step 3	Consider an approximate rectangle whose base is same as that of the representative strip and length equal to $\|x\| = \|f(y)\|$.
Step 4	Find the area of the approximating rectangle as $\|x\|$ dy = $\|f(y)\|$ dy.
Step 5	Find the value of y say $y = c$ and $y = d$ within which the approximating rectangle can move veerticaally in the given region and form the integral $\int_c^d \|x\|$ dy.
Step 6	Evaluate the integral obtained in Step 5. The value of the integral obtained is the required area.

ALGORITHM TO FIND AREA BETWEEN TWO CURVES USING VERTICAL STRIPS	
Step 1	Draw the given curves y = f (x) and y = g (x) and the vertical lines x = a and x = b.
Step 2	Identify the region included between the curves and the vertical lines drawn in Step 1.
Step 3	Take an arbitrary point P (x, y) on one of the curves say y = f (x) and draw a vertical line through P to meet the other curve at Q (x, y_2). Clearly y_1 = f (x) and y_2 = g(x).
Step 4	Draw a vertical approximately rectangle of width dx, height = $\|(y_1 - y_2)\| = \|f(x) - g(x)\|$ such that P (x, y_1) and Q (x, y_2) are the midpoints of the horizontal sides.
Step 5	Find the area of the approximating rectangle drawn in Step 4 such that Δ a = $\|f(x) - g(x)\|$ dx.
Step 6	Use the formula A = $\int_a^b \|f(x) - g(x)\|$ dx to find the area of the region between y = f (x), y = g (x) and on the left and right of the vertical lines x = a and x = b respectively.

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DIFFERENTIAL EQUATIONS

1. DIFFERENTIAL EQUATION is an equation containing an independent variable, a dependent variable and the derivatives of the dependent variable.
2. ORDER OF THE DIFFERENTIAL EQUATION is the order (2nd order derivative, 3rd order derivative, etc.) of the highest order derivative occurring in a differential equation.
3. DEGREE OF THE DIFFERENTIAL EQUATION is the power of the highest order derivative occurring in a differential equation.

NOTE
In case of differential equations involving one or more terms of the form $e^{dy/dx}$, log $\frac{dy}{dx}$, sin $\frac{dy}{dx}$, etc. the degree is not defined.

ALGORITHM TO FORM A DIFFERENTIAL EQUATION	
Step 1	Write the given equation involving the independent variable (say x) and the dependent variable (say y) and the arbitrary constants.
Step 2	Obtain the number of arbitrary constants. Let us assume there are n arbitrary constants.
Step 3	Differentiate the equation in Step 1 n times with respect to x.
Step 4	Eliminate the arbitrary constants with the help of n equations involving differential coefficients.
	The equation obtained above is the differential equation.

4. SOLUTION OF A DIFFERENTIAL EQUATION is a relation y = f(x) + C which satisfies a given differential equation.
5. GENERAL SOLUTION OF A DIFFERENTIAL EQUATION is that solution which contains as many arbitrary constants as the order of the differential equation.
6. PARTICULAR SOLUTION OF A DIFFERENTIAL EQUATION is the solution obtained by giving particular values to the arbitrary constants in the general solution of a differential equation.
7. METHODS OF SOLVING DIFFERENT TYPES OF DIFFERENTIAL EQUATIONS
   (a) If the differential equation is of the type $\frac{dy}{dx} = f(x)$, write $dy = f(x)\,dx$
   Integrating both sides obtain $\int dy = \int f(x)\,dx + C$, to get a general solution of the differential equation.
   (b) If the differential equation is of the type $\frac{dy}{dx} = f(y)$, write $\frac{dx}{dy} = \frac{1}{f(y)}$ provided that $f(y) \neq 0 \Rightarrow dx = \frac{1}{f(y)}\,dy$
   Integrating both sides obtain $\int dx = \int \frac{1}{f(y)}\,dy + C$, to get a general solution of the differential equation.
   (c) If the differential equation is of the type $f(x)\,dx = g(y)\,dy$, then simply integrate both sides to get
   $\int f(x)\,dx = \int g(y)\,dy + C$
8. HOMOGENEOUS FUNCTION of degree n is a function $f(x, y)$ in x and y if the degree of each term is n.
9. HOMOGENEOUS DIFFERENTIAL EQUATION is a differential equation that contains a function and a differentiation and all of its terms have the same degree.

ALGORITHM TO SOLVE A HOMOGENEOUS DIFFERENTIAL EQUATION	
Step 1	Let the differential equation be $\frac{dy}{dx} = \frac{f(x,\ y)}{g(x,\ y)}$
Step 2	Put y = vx and differentiate to get $\frac{dy}{dx} = v + x\frac{dv}{dx}$
Step 3	Put the values of y and $\frac{dy}{dx}$ obtained in Step 2 in the equation of Step 1 to get $v + x\frac{dv}{dx} = F(v)$.
Step 4	Shift v to RHS and separate the variables v and x.
Step 5	Integrate both sides to obtain the solution in terms of v and x.
Step 6	Replace v by $\frac{y}{x}$ in the solution obtained in Step 5 to get the solution in terms of x and y.

10. LINEAR DIFFERENTIAL EQUATION is that in which the dependent variable and its derivative appear only in first degree.

ALGORITHM TO SOLVE A LINEAR DIFFERENTIAL EQUATION	
Step 1	Write the differential equation in the form $\frac{dy}{dx} + Py = Q$
Step 2	Obtain the values of P and Q
Step 3	Find the integrating factor (I. F) by the formula I. F = $e^{\int P\,dx}$
Step 4	Now the required solution can be found by using the formula y x I. F. = $\int$(Q x I. F) dx + C
Step 5	Integrate the equation obtained in Step 4 with respect to x.

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VECTOR ALGEBRA

1. VECTOR is a quantity that has magnitude as well as direction.
2. SUPPORT is the line of unlimited length of $\overrightarrow{AB}$ where AB is a segment.
3. SENSE of $\overrightarrow{AB}$ is from A to B and that of $\overrightarrow{BA}$ is from B to A.
4. POSITION VECTOR OF A POINT is a vector that symbolizes the location of any point with reference to the origin.
5. UNIT VECTOR is a vector, $\vec{a}$, when $|\vec{a}| = 1$. It is denoted by $\hat{a}$ read as 'a cap'.
6. ZERO OR NULL VECTOR is a vector whose initial and terminal points coincide. It is denoted by $\vec{0}$.
7. EQUAL VECTORS are two vectors $\vec{a}$ and $\vec{b}$ which have the same magnitude and are in the same directions regardless of their initial points.
8. COINITIAL VECTORS are those vectors having the same initial point.
9. COLLINEAR VECTORS OR PARALLEL VECTORS are those vectors having the same or parallel supports.
10. LIKE VECTORS are vectors having the same directions.
11. UNLIKE VECTORS are vectors having opposite directions.
12. FREE VECTORS are those vectors whose initial point is not specified.
13. LOCALIZED VECTOR is a vector drawn parallel to a given vector through a specified point as the initial point.
14. COTERMINOUS VECTORS are those vectors which have the same terminal point.
15. NEGATIVE OF A VECTOR is a vector having the same magnitude as that of a given vector $\vec{a}$ but is in the opposite direction. It is denoted by $-\vec{a}$. $\overrightarrow{AB} = \vec{a}$ then $\overrightarrow{BA} = -\vec{a}$.
16. RECIPROCAL OF A VECTOR is a vector having the same direction as that of a given vector $\vec{a}$ but whose magnitude is the reciprocal of the given vector $\vec{a}$ and is denoted by $\vec{a}^{-1}$.
17. COPLANAR VECTORS are three or more vectors lying in the same plane or parallel to the same plane.
18. PARALLELOGRAM LAW OF ADDITION OF VECTORS states that if two vectors $\vec{a}$ and $\vec{b}$ are represented in magnitude and direction by the adjacent sides of a parallelogram, then their sum $\vec{c}$ is represented by the diagonal of the parallelogram which is coinitial with the given vectors.
19. TRIANGLE LAW OF ADDITION OF VECTORS states that if two vectors are represented in magnitude and direction by two sides of a triangle, then their sum is represented by the third side taken in the reverse order.

PROPERTIES OF VECTORS		
1	Commutative Property for Addition	$\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2	Associative Property of Addition	$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
3	Existence of Additive Identity	$\vec{a} + \vec{0} = \vec{a}$
4	Existence of Additive Inverse	$\vec{a} + (-\vec{a}) = \vec{0}$
5	Difference of 2 vectors	$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$
6	Scalar Multiplication of a vector	$k\ \vec{a} = \|k\,\vec{a}\|$
7		$m(-\vec{a}) = -(m\vec{a})$
8		$(-m)(-\vec{a}) = m\,\vec{a}$
9		$m(n\,\vec{a}) = (mn)\,\vec{a}$
10	Distributive Property	$(m + n)\,\vec{a} = m\,\vec{a} + n\,\vec{a}$
11	Distributive Property	$m(\vec{a} + \vec{b}) = m\,\vec{a} + m\,\vec{a}$

NOTE : $\overrightarrow{AB}$ = (Position vector of B) – (Position vector of A)

20. SECTION FORMULA (INTERNAL) : If A and B are two points with position vectors $\vec{a}$ and $\vec{b}$ respectively and if C is a point dividing AB internally in the ratio m : n then the position vector of C is $\frac{m\vec{b} + n\vec{a}}{m + n}$.
21. SECTION FORMULA (EXTERNAL) : If A and B are two points with position vectors $\vec{a}$ and $\vec{b}$ respectively and if C is a point dividing AB externally in the ratio m : n then the position vector of C is $\frac{m\vec{b} - n\vec{a}}{m - n}$.
22. MIDPOINT FORMULA : If A and B are two points with position vectors $\vec{a}$ and $\vec{b}$ respectively and if C is a point bisecting AB then the position vector of C is $\frac{\vec{a} + \vec{b}}{2}$.
23. CENTROID FORMULA : $\frac{\vec{a} + \vec{b} + \vec{c}}{3}$.
24. COLLINEARITY OF VECTORS : Two non – zero vectors $\vec{a}$ and $\vec{b}$ are collinear if $x\vec{a} + y\vec{b} = 0$
25. CCOPLANAR VECTORS : Three non – zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar if $l\vec{a} + m\vec{b} + n\vec{c} = 0$

COMPONENTS OF A VECTOR IN TWO DIMENSIONS	
1	$\overrightarrow{AB}$ = x$\hat{i}$ + y$\hat{j}$
2	$\lvert\overrightarrow{AB}\rvert = \sqrt{x^2 + y^2}$
3	Component of $\overrightarrow{AB}$ along x – axis is vector x$\hat{i}$ and its magnitude is $\lvert x\rvert$
4	Component of $\overrightarrow{AB}$ along y – axis is vector y$\hat{j}$ and its magnitude is $\lvert y\rvert$
5	If A (x_1, y_1) and B (x_2, y_2) then $\lvert\overrightarrow{AB}\rvert = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
6	If $\vec{a} = a_1\hat{i} + a_2\hat{j}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j}$, then $\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j}$
7	If $\vec{a} = a_1\hat{i} + a_2\hat{j}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j}$, then $\vec{a} - \vec{b} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j}$
8	If $\vec{a} = a_1\hat{i} + a_2\hat{j}$ and m is a scalar quantity then m $\vec{a} = (ma_1)\hat{i} + (ma_2)\hat{j}$

COMPONENTS OF A VECTOR IN THREE DIMENSIONS	
1	$\overrightarrow{AB}$ = x$\hat{i}$ + y$\hat{j}$ + z$\hat{k}$
2	$\lvert\overrightarrow{AB}\rvert = \sqrt{x^2 + y^2 + z^2}$
3	Component of $\overrightarrow{AB}$ along x – axis is vector x$\hat{i}$ and its magnitude is $\lvert x\rvert$
4	Component of $\overrightarrow{AB}$ along y – axis is vector y$\hat{j}$ and its magnitude is $\lvert y\rvert$
5	Component of $\overrightarrow{AB}$ along z – axis is vector z$\hat{k}$ and its magnitude is $\lvert z\rvert$
6	If A (x_1, y_1, z_1) and B (x_2, y_2, z_2) then $\lvert\overrightarrow{AB}\rvert = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
7	If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$
8	If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a} - \vec{b} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}$
9	If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and m is a scalar quantity then m $\vec{a} = (ma_1)\hat{i} + (ma_2)\hat{j} + (ma_3)\hat{k}$

26. COMPONENTS OF A VECTOR are the parts that make up the vector. $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$
27. DIRECTION RATIO represents the components of a vector with respect to the three axis.
    For example if the vector $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$, then the direction ratios are a, b and c.
28. DIRECTION COSINE represents the angle subtended by vector with respect to the three axis.
    For example if the vector $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$, then the direction cosines are $\frac{a}{\sqrt{a^2 + b^2 + c^2}}$, $\frac{b}{\sqrt{a^2 + b^2 + c^2}}$ and $\frac{c}{\sqrt{a^2 + b^2 + c^2}}$
    If the direction cosines of a vector are l, m, n, where $l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$ then $(l^2 + m^2 + n^2) = 1$

IMPORTANT THEOREMS ON DIRECTION RATIOS AND DIRECTION COSINES	
Theorem 1	If $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$, then its direction cosines are $\frac{a}{\sqrt{a^2 + b^2 + c^2}}$, $\frac{b}{\sqrt{a^2 + b^2 + c^2}}$ and $\frac{c}{\sqrt{a^2 + b^2 + c^2}}$
Theorem 2	Direction ratios of the line segment joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are proportional to $(x_2 - x_1)$, $(y_2 - y_1)$ and $(z_2 - z_1)$
Theorem 3	Two parallel vectors have proportional direction ratios. If vectors $\vec{a}$ and $\vec{b}$ are parallel then $\vec{b} = \lambda\,\vec{a}$
Theorem 4	If vector $\vec{r}$ has direction ratios proportional to (a, b, c) then $\vec{r} = \frac{\lvert\vec{r}\rvert}{\sqrt{a^2 + b^2 + c^2}}(a\hat{i} + b\hat{j} + c\hat{k})$
Theorem 5	If l, m, n, are the direction cosines of $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then its projections on the coordinate axis are $l\lvert\vec{r}\rvert$, $m\lvert\vec{r}\rvert$, $n\lvert\vec{r}\rvert$.

29. ANGLE BETWEEN TWO VECTORS : $\cos\theta = \frac{\vec{a}\bullet\vec{b}}{\lvert\vec{a}\rvert\,\lvert\vec{b}\rvert}$ or $\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}\sqrt{(b_1)^2 + (b_2)^2 + (b_3)^2}}$

NOTE : If $\vec{a}\bullet\vec{b} > 0$ then θ is an acute angle and if $\vec{a}\bullet\vec{b} < 0$ then θ is an obtuse angle.

30. DOT OR SCALAR PRODUCT OF TWO VECTORS : $\lvert\vec{a}\rvert\,\lvert\vec{b}\rvert\cos\theta$
31. LENGTH OF A VECTOR : $\lvert\vec{a}\rvert^2$
32. PROJECTION OF ONE VECTOR ($\vec{a}$) ON ANOTHER VECTOR ($\vec{b}$) : $\frac{\vec{a}\bullet\vec{b}}{\lvert\vec{b}\rvert}$ or $\vec{a}\bullet\hat{b}$
33. PROJECTION OF A VECTOR ($\vec{a}$) ON A LINE l : $\lvert\vec{a}\rvert\cos\theta$, where θ is the angle between the vector and the line.
34. IF 2 VECTORS $\vec{a}$ AND $\vec{b}$ ARE PERPENDICULAR TO EACH OTHER THEN $\vec{a}\bullet\vec{b} = 0$
35. IF 2 VECTORS $\vec{a}$ AND $\vec{b}$ ARE PARALLEL TO EACH OTHER THEN $\vec{a}\times\vec{b} = 0$
36. CAUCH SCHWARTZ INEQUALITY : $\lvert\vec{a}\bullet\vec{b}\rvert \le \lvert\vec{a}\rvert\,\lvert\vec{b}\rvert$
37. TRIANGLE INEQUALITY : $\lvert\vec{a} + \vec{b}\rvert \le \lvert\vec{a}\rvert + \lvert\vec{b}\rvert$

NOTE : If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors perpendicular to the coordinate axis then
1. $\hat{i}\bullet\hat{i} = \hat{j}\bullet\hat{j} = \hat{k}\bullet\hat{k} = 1$
2. $\hat{i}\bullet\hat{j} = \hat{j}\bullet\hat{i} = 0$, $\hat{j}\bullet\hat{k} = \hat{k}\bullet\hat{j} = 0$ and $\hat{i}\bullet\hat{k} = \hat{k}\bullet\hat{i} = 0$

37. VECTOR (CROSS) PRODUCT : $\vec{a}$ x $\vec{b}$ = $|\vec{a}|\,|\vec{b}|\sin\theta\,\hat{\eta}$, where θ is the angle between $\vec{a}$ and $\vec{b}$ and $\hat{\eta}$ is a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$ and $\hat{\eta}$ form a right handed system.
38. ANGLE BETWEEN TWO VECTORS : $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$
39. VECTOR PERPENDICULAR TO TWO GIVEN VECTORS : $\hat{\eta} = \frac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

PROPERTIES OF VECTOR PRODUCT		
Property 1	If $\vec{a}$ and $\vec{b}$ are any two vectors then their product is not commutative.	$\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$
Property 2	If $\vec{a}$ and $\vec{b}$ are any two vectors and m is a scalar, then the scalar can be multiplied with any one vector.	$m(\vec{a}\times\vec{b}) = m\vec{a}\times\vec{b} = \vec{a}\times m\vec{b}$
Property 3	If $\vec{a}$ and $\vec{b}$ are any two vectors and m, n are scalars, then	$m\vec{a}\times n\vec{b} = mn(\vec{a}\times\vec{b})$
Property 4	Distributive property of vector product over vector addition	$\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c})$
Property 5	Distributive property of vector product over vector subtraction	$\vec{a}\times(\vec{b}-\vec{c}) = (\vec{a}\times\vec{b}) - (\vec{a}\times\vec{c})$
Property 6	Vector product of two non – zero vectors is zero vector iff they are collinear.	$\vec{a}\times\vec{b} = \vec{0}$

40. VECTOR PRODUCT IN TERMS OF COMPONENTS : $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$
41. For any two vectors $\vec{a}$ and $\vec{b}$: $\left|\vec{a}\times\vec{b}\right|^2 = |\vec{a}|^2\left|\vec{b}\right|^2$
42. For any vector $\vec{a}$: $\left|\vec{a}\times\hat{i}\right|^2 + \left|\vec{a}\times\hat{j}\right|^2 + \left|\vec{a}\times\hat{k}\right|^2 = 2|\vec{a}|^2$
43. LENGTH OF PEREPENDICULAR FROM A ON BC = $\frac{|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}|}{|\vec{b}-\vec{c}|}$
44. VECTORS NORMAL TO THE PLANE OF TWO GIVEN VECTORS : $\frac{\lambda(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

SOME IMPORTANT RESULTS		
Result 1	Area of a parallelogram with adjacent sides $\vec{a}$ x $\vec{b}$	$\left|\vec{a}\times\vec{b}\right|$
Result 2	Area of a triangle with adjacent sides $\vec{a}$ x $\vec{b}$	$\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$ or $\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \frac{1}{2}\left|\overrightarrow{BC}\times\overrightarrow{BA}\right| = \frac{1}{2}\left|\overrightarrow{CB}\times\overrightarrow{CA}\right|$
Result 3	Area of a parallelogram with diagonals as $\vec{a}$ x $\vec{b}$	$\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$
Result 4	Area of a plane quadrilateral ABCD	$\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right|$

45. LAGRANGE'S IDENTITY : For any two vectors $\vec{a}$ x $\vec{b}$: $\left|\vec{a}\times\vec{b}\right|^2 = |\vec{a}|^2\left|\vec{b}\right|^2 - (\vec{a}\bullet\vec{b})^2$

THEOREMS ON SCALAR TRIPLE PRODUCTS	
Theorem 1	Volume of parallelepiped whose coterminous edges are $\vec{a}$, $\vec{b}$ and $\vec{c}$ is $(\vec{a}\times\vec{b})\bullet\vec{c}$
Theorem 2	For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $(\vec{a}\times\vec{b})\bullet\vec{c} = \vec{a}\bullet(\vec{b}\times\vec{c})$
Theorem 3	The scalar triple product of any three vectors remains unchanged if their cyclic order is not changed.
Theorem 4	The scalar triple product of any three vectors changes in sign but not in magnitude if their cyclic order is changed.
Theorem 5	The scalar triple product of any three vectors vanishes if any two of its vectors are equal.
Theorem 6	The scalar triple product of any three vectors vanishes if any two of its vectors are parallel or collinear.
Theorem 7	For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ we have $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$
Theorem 8	Necessary and sufficient conditions for three non – zero, non collinear vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ to be coplanar is $[\vec{a}\ \vec{b}\ \vec{c}] = 0$
Theorem 9	For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ the vectors $(\vec{a}-\vec{b})$, $(\vec{b}-\vec{c})$, $(\vec{c}-\vec{a})$ are coplanar.
Theorem 10	Any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar if and only if $(\vec{a}+\vec{b})$, $(\vec{b}+\vec{c})$, $(\vec{c}+\vec{a})$ are coplanar.
Theorem 11	For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$: $[\vec{a}\quad \vec{b}+\vec{c}\quad \vec{a}+\vec{b}+\vec{c}] = 0$
Theorem 12	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are position vectors of A, B, C then $(\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a})$ is a vector perpendicular to the plane of the triangle ABC.
Theorem 13	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are position vectors of A, B, C of Δ ABC then area of Δ ABC = $\frac{1}{2}\left|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}\right|$

VIJAYGIRI MATH CLASS

FORMULAE SHEET

THREE D GEOMETERY

SIGNS OF COORDINATES OF A POINT								
OCTANT → COORDINATE ↓	OXYZ	OX′YZ	OXY′Z	OX′Y′Z	OXYZ′	OX′YZ′	OXY′Z′	OX′Y′Z′
X	+	–	+	–	+	–	+	–
Y	+	+	–	–	+	+	–	–
Z	+	+	+	+	–	–	–	–

1. DISTANCE BETWEEN (x_1, y_1, z_1) and (x_2, y_2, z_2) = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
2. SECTION FORMULA = $\left(\frac{m x_2 + n x_1}{m+n}, \frac{m y_2 + n y_1}{m+n}, \frac{m z_2 + n z_1}{m+n}\right)$ (internal) or $\left(\frac{m x_2 - n x_1}{m-n}, \frac{m y_2 - n y_1}{m-n}, \frac{m z_2 - n z_1}{m-n}\right)$ (external)
3. MIDPOINT FORMULA = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
4. CENTROID FORMULA = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$
5. DIRECTION COSINES OF A LINE are $\boldsymbol{l = \cos\alpha, m = \cos\beta, n = \cos\gamma}$, where α, β and γ are the angles that the line makes with the x – axis, y – axis and z – axis respectively.

NOTE
1. Direction cosines of the x – axis are 1, 0, 0
2. Direction cosines of the y – axis are 0, 1, 0
3. Direction cosines of the z – axis are 0, 0, 1
4. $l^2 + m^2 + n^2 = 1$

6. DIRECTION RATIOS OF A LINE are any 3 numbers *a*, *b*, *c* which are proportional to the direction cosines *l*, *m*, *n* of the line. $\left[\frac{l}{a} = \frac{m}{b} = \frac{n}{c}\right]$ OR $\left[l = \frac{a}{\sqrt{a^2+b^2+c^2}}, m = \frac{b}{\sqrt{a^2+b^2+c^2}}, n = \frac{c}{\sqrt{a^2+b^2+c^2}}\right]$ OR $\left[\frac{a}{\cos\alpha} = \frac{b}{\cos\beta} = \frac{c}{\cos\gamma}\right]$
7. ANGLE BETWEEN TWO LINES : If θ is the angle between two lines l_1 and l_2 whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2 then :
   (i) $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$
   (ii) $\sin\theta = \sqrt{\sum(m_1 n_2 - m_2 n_1)^2}$
   (iii) lines l_1 and l_2 are perpendicular if $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$
   (iv) lines l_1 and l_2 are parallel if $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$
8. EQUATION OF A STRAIGHT LINE PASSING THROUGH A FIXED POINT IS
   (i) VECTOR FORM : $\vec{r} = \vec{a} + \lambda\vec{b}$ where $\vec{a}$ is the position vector of the point and the line is parallel to $\vec{b}$. $[\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}]$
   (ii) CARTESIAN FORM : $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ where the fixed point is (x_1, y_1, z_1) and having d. r's proportional to *a, b, c*
   (iii) CARTESIAN FORM : $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ where the fixed point is (x_1, y_1, z_1) and direction cosines are *l, m, n*
9. PARAMETRIC FORMS OF THE LINE $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ are $\mathbf{x = x_1 + \lambda a, y = y_1 + \lambda b, z = z_1 + \lambda c}$, where λ is the parameter.
10. COORDINATES OF ANY POINT ON THE LINE $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ are $(\mathbf{x_1 + \lambda a, y_1 + \lambda b, z_1 + \lambda c})$ where $\lambda \in R$.
11. EQUATION OF A STRAIGHT LINE PASSING THROUGH TWO GIVEN POINTS IS
    (i) VECTOR FORM : $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$ where the position vectors are $\vec{a}$ and $\vec{b}$
    (ii) CARTESIAN FORM : $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$ where the two given points (x_1, y_1, z_1) and (x_2, y_2, z_2)
12. COLLINEARITY CONDITION WHEN COORDINATES OF THREE GIVEN POINTS A (x_1, y_1, z_1), B (x_2, y_2, z_2) are C (x_3, y_3, z_3) is $\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1} = \frac{z_3 - z_1}{z_2 - z_1}$
13. COLLINEARITY CONDITION WHEN POSITION VECTORS $\vec{a}$, $\vec{b}$ and $\vec{c}$ OF THREE GIVEN POINTS are given is $\mathbf{p\vec{a} + q\vec{b} + r\vec{c} = \vec{0}}$ and p + q + r = 0, where p, q, r are scalars not all zero.
14. ANGLE BETWEEN TWO LINES
    (i) Whose vector forms are $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is $\cos\theta = \frac{\vec{b_1} \bullet \vec{b_2}}{|\vec{b_1}|\,|\vec{b_2}|}$
    (ii) Whose cartesian equations are $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ is $\cos\theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

NOTE : $\theta = \frac{\pi}{2}$ if $b_1 \bullet b_2 = 0$ or $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

15. COPLANAR LINES are those two lines which lie in the same plane. They are either parallel or intersecting.
16. SKEW LINES are those two lines which do not lie in the same plane. They are neither parallel nor intersecting.

ALGORITHM FOR EQUAATION OF A LINE PASSING THROUGH A GIVEN POINT & ⊥ TO 2 GIVEN LINES	
Step 1	Obtain the point through which the line passes. Let its position vector be $\vec{a}$
Step 2	Obtain the vectors parallel to the two given lines. Let the vectors be $\vec{b_1}$ and $\vec{b_2}$.
Step 3	Obtain $\vec{b_1} \times \vec{b_2}$.
Step 4	The vector equation of the given line is $\vec{r} = \vec{a_1} + \lambda(\vec{b_1} \times \vec{b_2})$.

ALGORITHM TO FIND THE POINT OF INTERSECTION OF TWO LINES (CARTESIAN FORM)	
Step 1	Let the two given lines be $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$
Step 2	Equating each line to constants λ and μ, write the coordinates of the general points as $(a_1\lambda + x_1, b_1\lambda + y_1, c_1\lambda + z_1)$ and $(a_2\mu + x_2, b_2\mu + y_2, c_2\mu + z_2)$
Step 3	If the two lines intersect then they have a common point. $\therefore a_1\lambda + x_1 = a_2\mu + x_2, b_1\lambda + y_1 = b_2\mu + y_2$ and $c_1\lambda + z_1 = c_2\mu + z_2$.
Step 4	Solve any two equations obtained in Step 3 in λ and μ. If the values of λ and μ satisfy the third equation, then the lines intersect otherwise they don't.
Step 5	To obtain the coordinates of the point of intersection substitute the values of λ or μ in the coordinates of the general points obtained in Step 2.

ALGORITHM TO FIND THE POINT OF INTERSECTION OF TWO LINES (VECTOR FORM)	
Step 1	Let the two lines be $\vec{r} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$ ……………………………….(i) And $\vec{r} = (a'_1\hat{i} + a'_2\hat{j} + a'_3\hat{k}) + \lambda(b'_1\hat{i} + b'_2\hat{j} + b'_3\hat{k})$ ………..…………………(ii)
Step 2	Since $\vec{r}$ in the equation of a line denotes the position vector of an arbitrary point on it, so the position vectors of arbitrary points on (i) and (ii) are $(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$ And $(a'_1\hat{i} + a'_2\hat{j} + a'_3\hat{k}) + \lambda(b'_1\hat{i} + b'_2\hat{j} + b'_3\hat{k})$
Step 3	If the two lines intersect then they have a common point. $\therefore (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = (a'_1\hat{i} + a'_2\hat{j} + a'_3\hat{k}) + \lambda(b'_1\hat{i} + b'_2\hat{j} + b'_3\hat{k})$.
Step 4	Solve any two equations obtained in Step 3 in λ and μ. If the values of λ and μ satisfy the third equation, then the lines intersect otherwise they don't.
Step 5	To obtain the coordinates of the point of intersection substitute the values of λ or μ in the coordinates of the general points obtained in Step 2.

17. Lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ will be intersecting if and only if $(\vec{b_1} \times \vec{b_2}) \bullet (\vec{a_2} - \vec{a_1}) = 0$

18. SHORTEST DISTANCE BETWEEN TWO SKEW LINES $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is **d =** $\left|\frac{(\vec{a_2} - \vec{a_1}) \bullet (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right|$

19. SHORTEST DISTANCE BETWEEN TWO PARALLEL LINES is **d =** $\left|\frac{(\vec{a_2} - \vec{a_1}) \times \vec{b}}{|\vec{b}|}\right|$

20. SHORTEST DISTANCE BETWEEN TWO SKEW LINES $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given as **S D =** $\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{D}}$ where D = $\sqrt{(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2}$

21. LENGTH OF PERPENDICULAR FROM A POINT P ($\vec{\alpha}$) ON A LINE $\vec{r} = \vec{a} + \lambda\vec{b}$ is **L =** $\sqrt{|\vec{\alpha} - \vec{a}|^2 - \left\{\frac{(\vec{\alpha} - \vec{a}) \bullet \vec{b}}{|\vec{b}|}\right\}^2}$

22. LENGTH OF PERPENDICULAR FROM A POINT P (x_1, y_1, z_1) ON A LINE $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$, where l, m and n are direction cosines of the line is $\sqrt{\{(a - x_1)^2 + (b - y_1)^2 + (c - z_1)^2\} - \{(a - x_1)l + (b - y_1)m + (c - z_1)n\}^2}$

PLANE

1. PLANE is a surface such that if any two points are taken on it then the line segment joining these points lies completely on the surface.
2. NORMAL TO A PLANE is a straight line that is perpendicular to every line lying on the plane.
3. GENERAL EQUATION OF A PLANE IS **ax + by + cz + d = 0** where a, b, c are the direction ratios normal to the plane.
4. VECTOR NORMAL to the plane ax + by + cz + d = 0 is $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$.
5. CARTESIAN EQUATION OF A PLANE PASSING THROUGH A POINT (x_1, y_1, z_1) and having direction ratios proportional to *a, b, c* for its normal is $[a(x - x_1) + b(y - y_1) + c(z - z_1)]$.

ALGORITHM FOR EQUATION OF A PLANE PASSING THROUGH A GIVEN POINT	
Step 1	Write the equation of a plane passing through (x_1, y_1, z_1) as $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$……………..(i)
Step 2	If the plane in Step 1 passes through (x_2, y_2, z_2) and (x_3, y_3, z_3), then $a(x_2 - x_1) + b(y_2 - y_1) + c(z_2 - z_1) = 0$ …………………(ii) and $a(x_3 - x_1) + b(y_3 - y_1) + c(z_3 - z_1) = 0$ …………………(iii)
Step 3	Solve equations (ii) and (iii) obtained in Step 2 for *a*, *b* and *c*.
Step 4	Substitute the values of *a*, *b* and *c* obtained in Step 3 in equation (i) to get the required plane.

NOTE

On eliminating *a*, *b* and *c* from equations (i), (ii) and (iii), we get $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$ as the equation of the plane passing through three given points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

6. EQUATION OF A PLANE IN THE INTERCEPT FORM is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ where a, b, c are the intercepts made by the plane on the x – axis, y – axis and z – axis respectively.

To determine the intercepts made by a plane with the coordinate axes, proceed as follows
1. For x – intercept, put y = 0 and z = 0 in the equation of the plane and obtain the value of x.
2. For y – intercept, put x = 0 and z = 0 in the equation of the plane and obtain the value of y.
3. For z – intercept, put x = 0 and y = 0 in the equation of the plane and obtain the value of z.

7. VECTOR EQUATION OF A PLANE PASSING THROUGH A GIVEN POINT HAVING POSITION VECTOR $\vec{a}$ AND NORMAL TO VECTOR $\vec{n}$ IS $(\vec{r} - \vec{a}) \bullet \vec{n} = 0$ or $\vec{r} \bullet \vec{n} = \vec{a} \bullet \vec{n}$
8. EQUATION OF A PLANE PASSING THROUGH THREE GIVEN POINTS IS
   (i) CARTESIAN FORM : $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$, when the points are A (x_1, y_1, z_1), B (x_2, y_2, z_2) and C (x_3, y_3, z_3)
   (ii) VECTOR FORM : $[\vec{r}\ \vec{a}\ \vec{b}] + [\vec{r}\ \vec{b}\ \vec{c}] + [\vec{r}\ \vec{c}\ \vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]$
9. THE VECTOR EQUATION OF A PLANE PASSING THROUGH POINTS HAVING POSITION VECTORS $\vec{a}$, $\vec{b}$ and $\vec{c}$ is
   (i) PARAMETRIC FORM : $\vec{r} = (1 - m - n)\,\vec{a} + \lambda\vec{b} + \mu\vec{c}$.
   (ii) NON PARAMETRIC FORM : $\vec{r} \bullet (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = \vec{a} \bullet (\vec{b} \times \vec{c})$.
10. EQUATION OF A PLANE AT A DISTANCE p FROM THE ORIGIN AND PERPENDICULAR TO $\hat{n}$
    (i) VECTOR FORM : $\vec{r} \bullet \hat{n} = p$
    (ii) CARTESIAN FORM : $lx + my + nz = p$, where *l*, *m*, *n* are the direction cosines of it's normal.
11. CARTESIAN EQUATION OF A NORMAL TO THE PLANE $lx + my + nz = p$ FROM THE ORIGIN ARE $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$

ALGORITHM TO REDUCE CARTESIAN FORM TO NORMAL	
Step 1	Keep the terms containing x, y and z on the LHS and shift the constant term to the RHS.
Step 2	If the constant term on RHS is not positive, make it positive by multiplying both sides by – 1.
Step 3	Divide each term on both sides by $\sqrt{a^2 + b^2 + c^2}$, where *a*, *b* and *c* are the coefficients of x, y and z respectively.
Step 4	The coefficients of x, y and z in the equation so obtained will be the direction cosines of the normal to the plane and the RHS will be the distance of the plane from the origin.

12. ANGLE θ BETWEEN TWO PLANES IS

(i) VECTOR FORM : $\cos\theta = \frac{\vec{n_1} \bullet \vec{n_2}}{|\vec{n_1}|\,|\vec{n_2}|}$ when the vectors are $\vec{r} \bullet \vec{n_1} = d_1$ and $\vec{r} \bullet \vec{n_2} = d_2$

(ii) CARTESIAN FORM : $\cos\theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$ when the planes are $a_1 x + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$

NOTE :
1. The planes are parallel to each other if $\vec{n_1}$ is parallel to $\vec{n_2}$.
2. The planes are perpendicular to each other if $\vec{n_1} \bullet \vec{n_2} = 0$.
3. The planes are parallel to each other if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
4. The planes are perpendicular to each other if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$.

13. EQUATION OF A PLANE PASSING THROUGH A POINT HAVING POSITION VECTOR $\vec{a}$ AND PARALLEL TO VECTORS $\vec{b}$ AND $\vec{c}$ IS

(i) PARAMETRIC FORM : $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ where λ and μ are parameters OR $\vec{r} \bullet (\vec{b} \times \vec{c}) = \vec{a} \bullet (\vec{b} \times \vec{c})$

(ii) NON PARAMETRIC FORM : $(\vec{r} - \vec{a}) \bullet (\vec{b} \times \vec{c}) = 0$ OR $[\vec{r}\ \ \vec{b}\ \ \vec{c}] = [\vec{a}\ \ \vec{b}\ \ \vec{c}]$

14. EQUATION OF A PLANE PASSING THROUGH THE INTERSECTION OF TWO PLANES IS

(i) VECTOR FORM : $(\vec{r} - \vec{a}) \bullet (\vec{b} \times \vec{c}) = 0$ OR $[\vec{r}\ \ \vec{b}\ \ \vec{c}] = [\vec{a}\ \ \vec{b}\ \ \vec{c}]$ when the planes are $\vec{r} \bullet \vec{n_1} = d_1$ and $\vec{r} \bullet \vec{n_2} = d_2$ and λ is an arbitrary constant.

(ii) CARTESIAN FORM : $(a_1 x + b_1 y + c_1 z + d_1) + \lambda(a_2 x + b_2 y + c_2 z + d_2) = 0$, when the planes are $a_1 x + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$ and λ is a constant.

15. LENGTH OF A PERPENDICULAR FROM A POINT TO THE PLANE $\vec{r} \bullet \vec{n} = d$ IS

(i) VECTOR FORM : $p = \frac{|\vec{a} \bullet \vec{n} - d|}{|\vec{n}|}$ when the position vector of the point is $\vec{a}$

(ii) CARTESIAN FORM : $p = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ when the point is (x_1, y_1, z_1) and the plane is $ax + by + cz + d = 0$

NOTE :
1. Coordinates (α, β, γ) of the foot of the perpendicular are given by $\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{\gamma - z_1}{c} = -\left(\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}\right)$
2. Coordinates (α, β, γ) of the image of point in the plane are given by $\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{\gamma - z_1}{c} = -2\left(\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}\right)$

16. ANGLE BETWEEN A LINE AND THE PLANE IS

(i) VECTOR FORM : $\sin\theta = \frac{\vec{b} \bullet \vec{n}}{|\vec{b}|\,|\vec{n}|}$ when the line is $\vec{r} = \vec{a} + \lambda\vec{b}$ and the plane is $\vec{r} \bullet \vec{n} = d$

(ii) CARTESIAN FORM : $\sin\theta = \frac{al + bm + cn}{\sqrt{a^2 + b^2 + c^2}\sqrt{l^2 + m^2 + n^2}}$ when the line is $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ and the plane is $ax + by + cz + d = 0$

17. CONDITION OF PERPENDICULARITY : $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$

[The line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is perpendicular to the plane $a_2x + b_2y + c_2z = 0$ only if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$]

18. CONDITION OF PARALLELISM : $al + bm + cn = 0$

[The line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to the plane $a_2x + b_2y + c_2z = 0$ only if $[a_1a_2 + b_1b_2 + c_1c_2 = 0]$]

19. CONDITIONS FOR A LINE TO LIE IN A PLANE

(i) VECTOR FORM : If the line $\vec{r} = \vec{a} + \lambda\vec{b}$ lies in the plane $\vec{r} \bullet \vec{n} = d$ then $\vec{b} \bullet \vec{n} = 0$ and $\vec{a} \bullet \vec{n} = d$

(ii) CARTESIAN FORM : If the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ lies in the plane $ax + by + cz + d = 0$ then $ax_1 + by_1 + cz_1 + d_1 = 0$ and $al + bm + cn = 0$

20. CONDITIONS OF COPLANARITY OF TWO LINES AND EQUATION OF THE PLANE CONTAINING THEM

(i) VECTOR FORM : If the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are coplanar $\vec{r} \bullet (\vec{b_1} \times \vec{b_2}) = \vec{a_2} \bullet (\vec{b_1} \times \vec{b_2})$ and the equation of the plane containing them is $\vec{r} \bullet (\vec{b_1} \times \vec{b_2}) = \vec{a_1} \bullet (\vec{b_1} \times \vec{b_2})$

(ii) CARTESIAN FORM : If the lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ are coplanar then

$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$ and the equation of the plane is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

FORMULAE SHEET

LINEAR PROGRAMMING

1. LINEAR CONSTRAINTS are the limitations on the resources under which optimization is to accomplished and are expressed as inequations.
2. OBJECTIVE FUNCTION is the primary purpose of the formulation of a linear function of the involved variables needed to maximize or minimize subject to given constraints. It is always non negative.
3. OPTIMAL VALUE of an objective function is its maximum or minimum value.
4. FEASIBLE SOLUTION is a set of values of the variables satisfying all the constraints and the non – negativity restrictions.
5. FEASIBLE REGION is the common region determined by all the constraints and every point in this region is a feasible solution.
6. OPTIMAL SOLUTION is a feasible solution that leads to the optimal value of an objective function.
7. OPTIMIZATION TECHNIQUES are the processes of obtaining the optimal values of a system of inequations.
8. CONVEX SET is that set if every point on the line segment joining any two points in it lies in it.
9. NON–NEGATIVITY RESTRICTIONS are the constraints which describe that the variables in a LPP are nonnegative.

ALGORITHM TO FORMULATE A LINEAR PROGRAMMING PROBLEM (LPP)	
Step 1	Identify the unknown in the given LPP and denote them by variables like x and y.
Step 2	Formulate the objective functions in terms of x and y.
Step 3	Translate all the constraints in the form of linear inequations.
Step 4	Solve these inequations simultaneously.
Step 5	Mark the common area by shading it. This is the feasible region.
Step 6	Find the coordinates of all the vertices of this feasible region.
Step 7	Find the value of the objective function at each vertex of the feasible region.
Step 8	Find the values of x and y for which the objective function Z = ax + by has maximum or minimum value as the case maybe,

ALGORITHM TO SOLVE A LINEAR PROGRAMMING PROBLEM (LPP) BY CORNER POINT METHOD	
Step 1	Formulate the LPP in mathematical form.
Step 2	Convert all inequations into equations and draw their graphs. By putting y = 0 get a point on the x – axis and by putting x = 0, get a point on the y – axis. Joining these points gives a graph of the equation.
Step 3	Determine the region represented by each inequation. For a valid statement the region containing the origin is the region represented by the inequation. Otherwise the region not containing the origin is the region represented by the inequation.
Step 4	Obtain the region in the *xy* – plane containing all points that simultaneously satisfy all constraints including non – negativity restrictions. The polygonal region so obtained is the feasible region.
Step 5	Determine the coordinates of all the vertices (corner points) of the convex polygon.
Step 7	Obtain the value of the objective function at each vertex of the convex polygon.
Step 8	The point where the objective function attains the optimal value (maximum or minimum) is the optimal solution of the given LPP.

NOTE :

1. If the feasible region of the LPP is bounded as a convex polygon then the objective function **Z = *ax* + *by*** has both a maximum value ***M*** and a minimum value ***m*** and each of these values occurs at a corner point of this polygon.
2. If the feasible region of the LPP is un – bounded then we find the values of the objective function **Z = *ax* + *by*** at each corner point of the feasible region.

ALGORITHM TO SOLVE A LINEAR PROGRAMMING PROBLEM (LPP) BY ISO PROFIT LINES	
Step 1	Formulate the LPP in mathematical form.
Step 2	Obtain the region in the *xy* – plane containing all points that simultaneously satisfy all constraints including non – negativity restrictions. The polygonal region so obtained is the convex set of all feasible solutions of the LPP.
Step 3	Determine the coordinates of all the vertices (corner points) of the feasible region obtained in Step 2.
Step 4	Give some convenient value to Z and draw the line so obtained in the *xy* – plane.
Step 5	If the objective function is of maximization type then draw lines parallel to the line in Step 4 and obtain a line which is farthest from the origin and has at least one point common to the feasible region.
Step 6	If the objective function is of minimization type then draw lines parallel to the line in Step 4 and obtain a line which is nearest to the origin and has at least one point common to the feasible region.
Step 7	Find the coordinates of the common point obtained in Step 5 or in Step 6. The point so obtained determines the optimal solution and the value of the objective function at these point gives the optimal solution.

FORMULAE SHEET

PROBABILITY

1. RANDOM EXPERIMENT is an experiment which when repeated under identical conditions does not produce the same outcome every time but it is one of the several possible outcomes in a trial.
2. ELEMENTARY EVENT is each of the outcome in a trial when a random experiment is performed.
3. SAMPLE SPACE is set of all possible outcomes of a random experiment.
4. NUMBER OF SAMPLE SPACE WHEN A COIN IS TOSSED = $\mathbf{2^n}$, where n is the number of tosses.
5. NUMBER OF SAMPLE SPACE WHEN A DIE IS ROLLED = $\mathbf{6^n}$, where n is the number of rolls.

REMEMBER
1. Number of extra days in a normal year = **1**
2. Number of extra days in a leap year = **2**
3. Number of cards in a deck = **52 (26 of Black color, i.e. Clubs & Spades and 26 of Red color i.e. Hearts & Diamonds)**
4. Number of Picture cards in a deck = **12 (6 of Black color and 6 of Red color)**
(2 kings, 2 queens and 2 jacks of Black and 2 kings, 2 queens and 2 jacks of Red)

6. EVENT is a subset of the sample space associated with a random experiment.
7. CERTAIN EVENT is an event associated with a random experiment such that it always occurs whenever the experiment is performed.
8. IMPOSSIBLE EVENT is an event associated with a random experiment such that it never occurs whenever the experiment is performed.
9. COMPOUND EVENT is an event associated with a random experiment if it is the disjoint of two or more elementary events.
10. MUTUALLY EXCLUSIVE EVENTS are those two or more events associated with a random experiment such that the occurrence of one prevents the occurrence of other. $\mathbf{P(A \cup B) = P(A) + P(B)}$
11. EXHAUSTIVE EVENTS are those two or more events associated with a random experiment such that their union is the sample space. $\mathbf{P(A) + P(B) = 1}$
12. INDEPENDENT EVENTS are those events where the occurrence or non – occurrence of one event does not the probability of the occurrence or non – occurrence of the other. $\mathbf{P(A \cap B) = P(A) \times P(B)}$ OR $\mathbf{P(A \mid B) = P(A)}$

NOTE
Mutually exclusive events are those of a sample space which have no common outcome, i.e. $A \cap B = \phi$
Independent events are those where the occurrence or non – occurrence of one does not affect the probability of the other.

13. PROBABIITY OF AN EVENT HAPPENING : $\mathbf{P(E) = \frac{n(E)}{n(S)}}$
14. PROBABIITY OF AN EVENT NOT HAPPENING : $\mathbf{P(\bar{E}) = 1 - P(E)}$
15. CONDITIONAL PROBABILITY is the occurrence of an event A given that event B has already occurred. It is written as $\mathbf{P(A \mid B)}$

NOTE
1. In defining $P(A \mid B)$, $P(B) \neq 0$. If $P(B) = 0$, then B is an impossible event and so the assumption that B has occurred becomes meaningless.
2. If the sample space S of a random experiment consists of equally likely outcomes then in calculating $P(A \mid B)$, we need to only calculate $P(A)$ with respect to reduce sample space B.
3. If the sample space S of a random experiment does not consist of equally likely outcomes, then $\mathbf{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$, $\mathbf{P(B) \neq 0}$.
4. $P(A \mid S) = \frac{P(A \cap S)}{P(S)} = \frac{P(A)}{1} = P(A)$
5. $P(A \mid A) = \frac{P(A \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$
6. $P(S \mid A) = \frac{P(S \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$

PROPERTIES OF CONDITIONAL PROBABILITY	
Property 1	The conditional probability of an event A given that B has occurred lies between 0 and 1.
Property 2	If A and B are any two events associated with a random experiment having sample space S and F is an event such that $P(F) \neq 0$, then a) $\mathbf{P(A \cup B \mid F) = P(A \mid F) + P(B \mid F) - P(A \cap B \mid F)}$ b) $\mathbf{P(A \cup B \mid F) = P(A \mid F) + P(B \mid F)}$ in case of disjoint events.

	DESCRIPTION OF AN EVENT	EQUIVALENT SET NOTATION
1	Not an event A	$\bar{A}$
2	Either event A or event B	$A \cup B$
3	Both events A and B	$A \cap B$
4	Event A but not event B	$A \cap \bar{B}$
5	Neither event A nor event B	$\bar{A} \cap \bar{B}$
6	At least one of event A or event B or event C	$A \cup B \cup C$
7	Exactly one of event A and event B	$(A \cap \bar{B}) \cup (\bar{A} \cap B)$
8	All three events A, B and C	$A \cap B \cap C$
9	Exactly two of the events A, B and C	$(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$

	SOME RESULTS ON PROBABILITY	
1	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
2	$P(A \cup B) = P(A) + P(B - A) = P(B) + P(A - B)$	
3	$P(A) = P(A - B) + P(A \cap B)$	
4	$P(B) = P(B - A) + P(A \cap B)$	
5	$P(A - B) = P(A \cap \bar{B})$	
6	$P(B - A) = P(\bar{A} - B)$	
7	$P(A) + P(B) = P(A - B) + P(B - A) + 2P(A \cap B)$	
8	$P(A \cup B) = P(A) + P(B)$	**[For mutually exclusive events]**
9	$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$	
10	$P(A \cap \bar{B}) = P(A) - P(A \cap B)$	**[Probability of only A occurring]**
11	$P(\bar{A} \cap B) = P(B) - P(A \cap B)$	**[Probability of only B occurring]**
12	$P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A) + P(B) - 2P(A \cap B)$	**[Probability of exactly one occurring]**
12	$P(A / B) = \frac{P(A \cap B)}{P(B)}$; $P(B) \neq 0$	**[Conditional Probability]**
14	$P(S / B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$	
15	$P(A \cap B) = P(B) \times P(A / B)$; $P(B) \neq 0$ OR $P(A \cap B) = P(A) \times P(B / A)$; $P(A) \neq 0$	
16	$P(A \cap B \cap C) = P(A) \times P(B / A) \times P[C / (A \cap B)]$	
17	$P(A \cup B) = 1 - P(\bar{A}) P(\bar{B})$	
18	$P[(A \cup B)/C] = P(A / C) + P(B / C) - P[(A \cap B)/C]$	
19	$P[(A \cap B)/C] = P(A/C) + P(B/C)$	
20	$P(\bar{A}/B) = 1 - P(A/B)$	
21	$P(A \cap B) = P(A) \times P(B)$	**[Independent Events]**
22	$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$	**[Independent Events]**
23	$P[(A \cup B)/E] = P(A/E) + P(B/E) - P[(A \cap B)/E]$	
24	$P[(A \cup B)/E] = P(A/E) + P(B/E)$	**[Disjoint sets]**

NOTE

1. Probability of an event occurring can never be less than 0 or more than 1.
2. If probability of an event occurring is 0, then it is an impossible event.
3. If probability of an event occurring is 1, then it is a sure shot event.

16. TOTAL PROBABIITY THEOREM states that if $A_1, A_2, A_3, \ldots, A_n$ are the events of the sample space S such that the probability of none of these events is equal to zero, then the probability of an event E occurring in such a sample space is : $\mathbf{P(E) = \sum_{i=1}^{n} P(A_i)\, P(E / A_i)}$

17. BAYES THEOREM states that if $A_1, A_2, A_3, \ldots, A_n$ are mutually exclusive events associated with a random experiment and E is any event associated with the experiment then $\mathbf{P(A_i \mid E) = \frac{P(A_i) P(E \mid A_i)}{\Sigma P(A_i) P(E \mid A_i)}}$, where i = 1, 2, 3, …., n

- The probability $P(A_i \mid E)$ means finding the probability of event A_i given that event E has already occurred.
- Probability A_i was already known so it is called a **PRIORI POSSIBILITY**.
- Probability $P(A_i \mid E)$ has to be calculated knowing that event E has happened so it is called a **POSTERIORI PROBABILITY**.

18. PROBABILITY DISTRIBUTION OF A RANDOM VARIABLE describes how the probabilities are distributed over a random variable

19. DISCRETE PROBABILITY DISTRIBUTION is that in which the random takes distinct values such as the outcome of a die roll. Examples include Binomial and Poisson Distributions.

20. CONTINUOUS PROBABILITY DISTRIBUTION is that in which the random variable takes any values within a certain range such as a person's height. Examples include Normal and Uniform Distributions.
21. BERNOULLI DISTRIBUTION is that which has only two possible outcomes, success (I) and failure (0).

CONDITIONS FOR PROBABILITY DISTRIBUTION
- Each probability must be between 0 and 1 **$[0 \le P(x) \le 1]$**
- The sum of all probabilities must be 1 **$[\Sigma P(x) = 1]$**

ALGORITHM FOR PROBABILITY DISTRIBUTION	
Step 1	Define and clearly state what the random variable represents in the experiment and what values it can take.
Step 2	Identify all potential values that the random variable can assume based on the experiment.
Step 3	For each possible value of the random variable, calculate the probability of that value occurring using the appropriate probability formula based on the experiment.
Step 4	Ensure that all calculated probabilities are between 0 and 1 and the sum of all probabilities is equal to 1.
Step 5	Organize the information in a table with the possible random values in one row (or column) and their corresponding probabilities in the next row (or column).

22. MEAN OF PROBABILITY DISTRIBUTION is defined as the sum of the product of each element with its probability. **$[\text{Mean } \mu = \Sigma p_i x_i = p_1 x_1 + p_2 x_2 + p_3 x_3 + \ldots + p_n x_n]$**
23. VARIANCE OF PROBABILITY DISTRIBUTION is the sum of the product of the squared difference between each element and the mean by its probability. **$[\text{Variance } \sigma^2 = \Sigma p_i (x_i - \mu)^2 = \Sigma p_i x_i^2 - \mu^2 = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2]$**
24. BINOMIAL DISTRIBUTION is a discrete probability distribution of the number of successes in a sequence of *n* independent experiments each having only two possible outcomes, success (with probability *p*) or failure (with probability *q*). **$P(X = r) = {}^nC_r\, p^r q^{n-r}$**

NOTE
- Chances of all *n* trials succeeding are **$P(X = n) = p^n$**
- Chances of all *n* trials failing are **$P(X = 0) = q^n$ or $(1 - p)^n$**
- Chances of at least one success are **$P(X \ge 1) = 1 - q^n$ or $1 - (1 - p)^n$**
- Chances of at least r successes are **$P(X \ge r) = \sum_{k=r}^{n} C_k^n\, p^k q^{n-k}$**
- Chances of at most r successes are **$P(X \le r) = \sum_{k=0}^{r} C_k^n\, p^k q^{n-k}$**

25. MEAN OF A BINOMIAL DISTRIBUTION is the product of the number of trials (*n*) and the probability of success (*p*). **$\mu = np$**
26. VARIANCE OF A BINOMIAL DISTRIBUTION is **$np(1 - p)$** OR **npq**